
A Bayesian model for filling of a product to reduce risk of being OOS in presence of uncertainty.

■ Context

- A manufacturing site receive a (**concentrated**) bulk Drug Substance
- **Dilution** with a buffer has to be made to obtain the Drug Product ready to be filled in vials

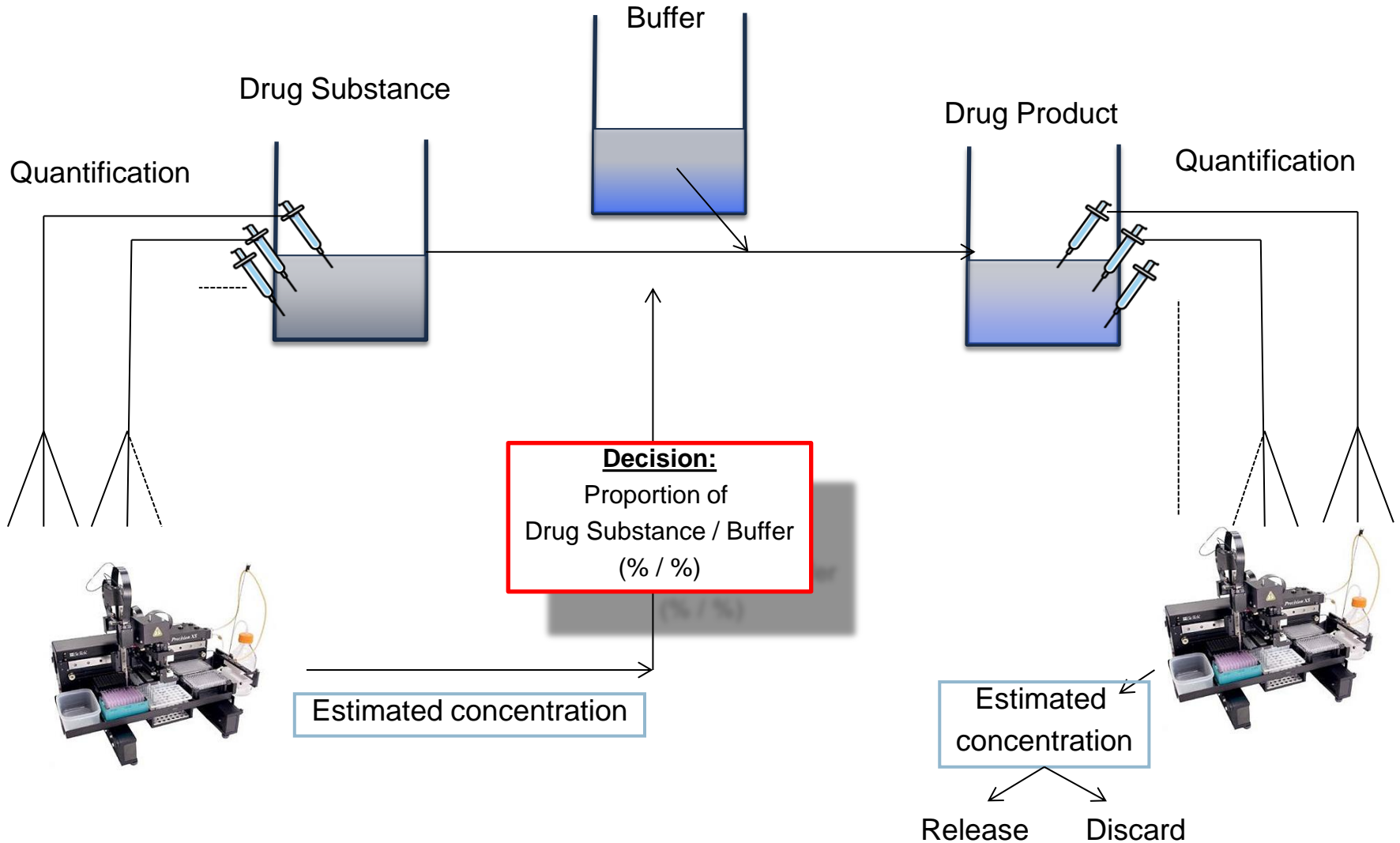
■ Problem

- The **quantification** of both the Drug Substance and Drug Product is subject to uncertainty
- Assume the **specification** release limit for Drug Product is $LSL=2$ mg/mL
If the concentration $> LSL$, Drug Product \rightarrow discarded

■ Question

- What is the (safe) dilution to ensure the Drug Product is within specification ?
- What is the **format** of the assays to obtain satisfactory reportable results about content ?

Dilution



- Start with the end

What is the very objective of the assays ?

- To provide results used to make important **decisions**

- Release of a production batch after dilution
- Optimization of a process (dilution protocol)
- Etc...

- What matters are the **results** produced by the assays, not the assays!

- *E.g. dilution will be decided based on the results obtained*
- *E.g. batch will be released based on the results obtained...*

- It must provide, in its future use, **quality product**
 - e.g. during routine
- According to specifications
- Whatever future conditions of use, that are not always perfectly controlled
 - Then, results should be **not sensitive** to minor changes
 - e.g. dilution not perfect, failed assay
- This is Quality by Design
 - The way the assays are developed leads to know quality & risks

■ Simulations

- Idea: test the dilution with different formats, at different levels of (mean) concentration for the Drug Substance
- Remember that neither the Drug Substance nor the Drug Product concentrations are known with certainty
- Thus, rely on the estimated posterior predictive distribution of the concentrations

■ Question

- What are the guarantees that, from an estimated concentration of the **Drug Substance** over a certain number of series and replicates, the resulting diluted **Drug Product** is within specification given an estimated concentration over a certain number of series and replicates ?

→ Design Space problem

- Over the **dilution** factor and the **format**

■ Viewed as a Design Space problem

– Optimization over the following factors

J_{DS} : the number of (independent) series for Drug Substance measurements

n_{DS} : the number of replicates/series for Drug Substance measurements

J_{DP} : the number of (independent) series for Drug Product measurements

n_{DP} : the number of replicates/series for Drug Product measurements

d : the dilution to apply (% of Drug Substance)

conc_{DS} : the true concentration of Drug Substance (not to be optimized)

– CQA

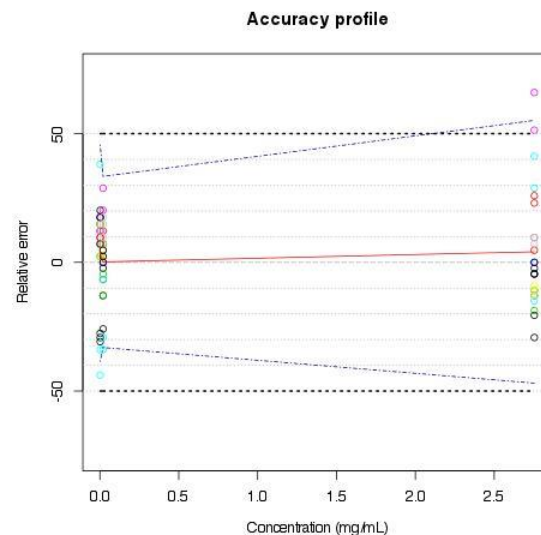
\bar{y}_{DP} : Reportable results of the concentration of Drug Product

– Specifications

$$\bar{y}_{DP} > 2 \text{ (mg/mL)}$$

$$2 < \bar{y}_{DP} < 2.4 \text{ (mg/mL), if possible}$$

- Precision of the assay is first provided with qualification data



9 series
3 replicates
3 concentration levels

- Assuming the level 2.7 mg/mL is the closest to the targeted concentration, and precision is homogenous among levels
 - The same precision will be used for any concentration levels of Bulk and Drug product
 - This assumption is useful as these are so far the only available data

■ Model description (for one concentration level)

$$y_{ij} = \mu + \alpha_j + \varepsilon_{ij}, \quad \text{with } \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2) \quad \text{and } \alpha_j \sim N(0, \sigma_\alpha^2)$$
$$j = 1, \dots, m, \quad i = 1, \dots, n_j, \quad n = \sum_{j=1}^m n_j.$$

■ Prior distributions for μ , σ_ε^2 , σ_α^2

– $p(\mu) = N(\mu_0, \tau_0)$ $p(\sigma_\varepsilon^2) = \text{gamma}(a, b)$ $p(\sigma_\alpha^2) = \text{gamma}(c, d)$

■ Example of BUGS model for unbalanced data

```
model{
  for(j in 1:m){
    for(i in n[j]:(n[j+1]-1)){
      y[i] ~ dnorm(a[j], tau.e) #likelihood
    }
    a[j] ~ dnorm(mu, tau.a)
  }

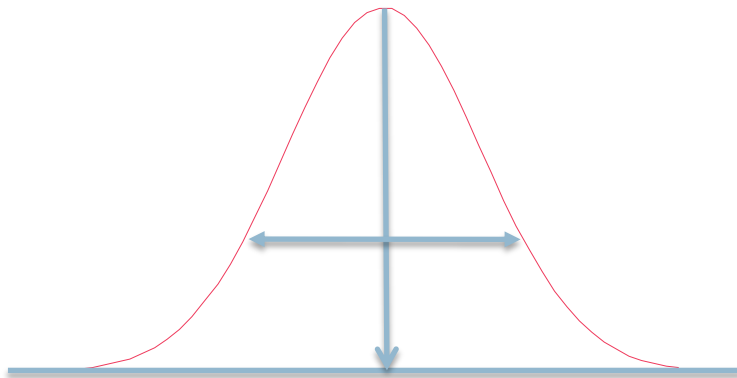
  #flat prior distributions
  mu ~ dnorm(0, 0.0001)
  tau.a ~ dgamma(0.0001, 0.0001)
  tau.e ~ dgamma(0.0001, 0.0001)

  #convert precision into variance
  sigma2.e <- 1/tau.e
  sigma2.a <- 1/tau.a
}
```

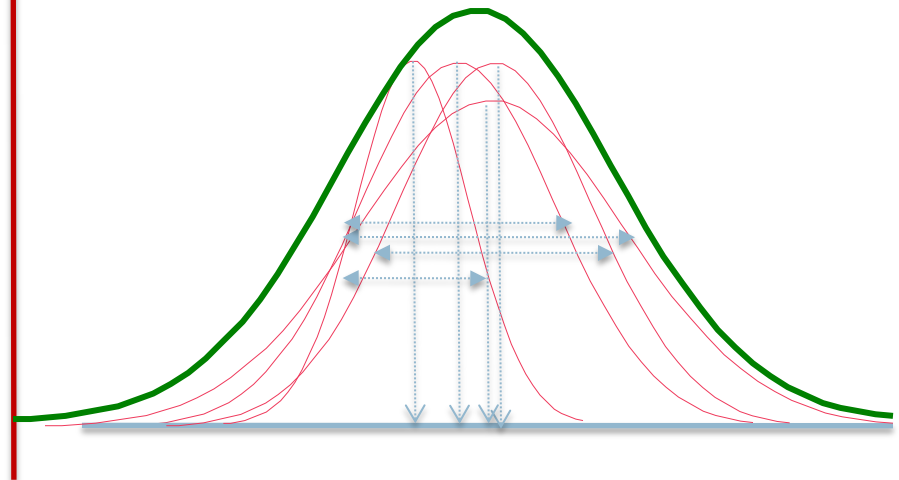
Output:

$$p(\mu, \tau_\alpha, \tau_\varepsilon \mid \text{data})$$

the “new observations” are drawn from distribution “centered” on estimated location and dispersion parameters (treated as “true values”).



the uncertainty of parameter estimates (location and dispersion) is taken into account before drawing “new observations” from relevant distribution



To account for uncertainty of prediction under the variation of series and replicates

→ One way ANOVA random model

→ accounting for uncertainty of (mean and variances) parameter estimates

■ Prediction applied to this model

$$p(\tilde{y} \mid \text{data}) = \int_{\mu} \int_{\tau_{\alpha}} \int_{\tau_{\epsilon}} p(\tilde{y} \mid \mu, \tau_{\alpha}, \tau_{\epsilon}) \cdot p(\mu, \tau_{\alpha}, \tau_{\epsilon} \mid \text{data}) \cdot d\tau_{\epsilon} \cdot d\tau_{\alpha} \cdot d\mu$$

- Not solvable (see Mee's approximation, 1984)

■ Sampling scheme to obtain samples from the predictive distribution

For $s = 1$ to n^*

1. sample $(\mu^{(s)}, \tau_{\alpha}^{(s)}, \tau_{\epsilon}^{(s)})$ from $p(\mu, \tau_{\alpha}, \tau_{\epsilon} \mid \text{data})$, **(from BUGS output)**
2. sample $\tilde{\alpha}^{(s)}$ from $N(\mu^{(s)}, \sigma_{\alpha}^{2(s)})$, or from $N(\mu^{(s)}, 1/\tau_{\alpha}^{(s)})$, **(in R or SAS)**
3. sample $\tilde{y}^{(s)}$ from $N(\tilde{\alpha}^{(s)}, \sigma_{\epsilon}^{2(s)})$, or from $N(\tilde{\alpha}^{(s)}, 1/\tau_{\epsilon}^{(s)})$.

End

■ one simulation at one “operating condition”

1. Measure of the Drug Substance

Sample J_{DS} series ($\tilde{a}_{DS}^{(s)} | \text{data}$) following $N(\text{conc}_{DS} | \text{data}, \sigma_a | \text{data})$

uncertainty of the mean scaled to target concentration

- For each series s , sample n_{DS} measurements $N(\tilde{a}_{DS}^{(s)} | \text{data}, \sigma_e^{(s)} | \text{data})$
- Compute the grand mean (rep. result) of the $n_{DS} * J_{DS}$ measurements: \bar{y}_{DS}

2. For a given dilution d

- Compute the true but unknown concentration of Drug Product: $\text{conc}_{DS} * d$
- Compute the concentration one would obtain if mean_{DS} was true: $\bar{y}_{DS} * d$

3. Measure of the Drug Product

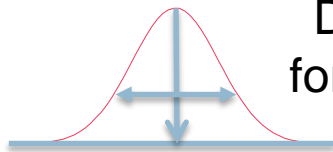
Sample J_{DP} series ($\tilde{a}_{DP}^{(s)} | \text{data}$) following $N(\text{conc}_{DS} * d | \text{data}, \sigma_a | \text{data})$

- For each series s , sample n_{DP} measurements $N(\tilde{a}_{DP}^{(s)} | \text{data}, \sigma_e^{(s)} | \text{data})$
- Compute the grand mean of the $n_{DP} * J_{DP}$ measurements: \bar{y}_{DP}

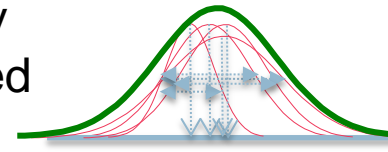
■ 1000 simulations this “operating condition”

Compute $P(\bar{y}_{DP} > 2 \ \& \ \text{conc}_{DS} * d > 2 | \text{data})$ using Monte - Carlo

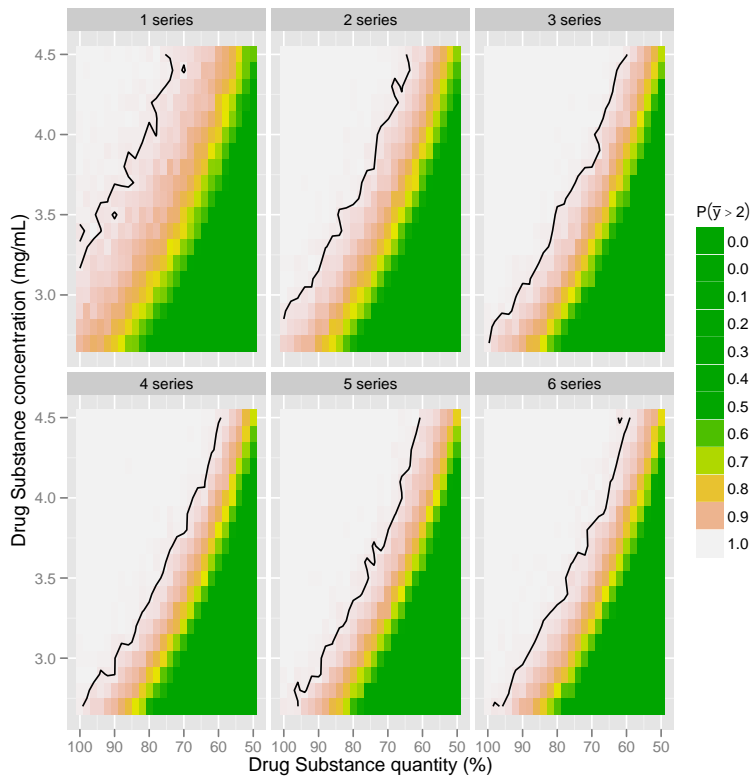
First results (process Design Space)



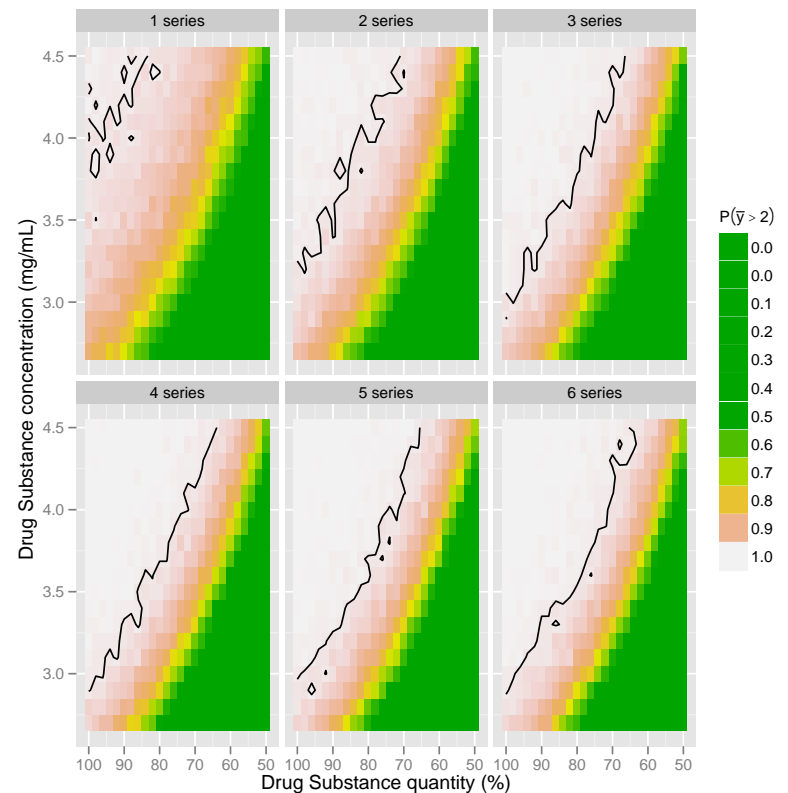
Draw a map of the probability
for every factor setting explored



Without parameter uncertainties



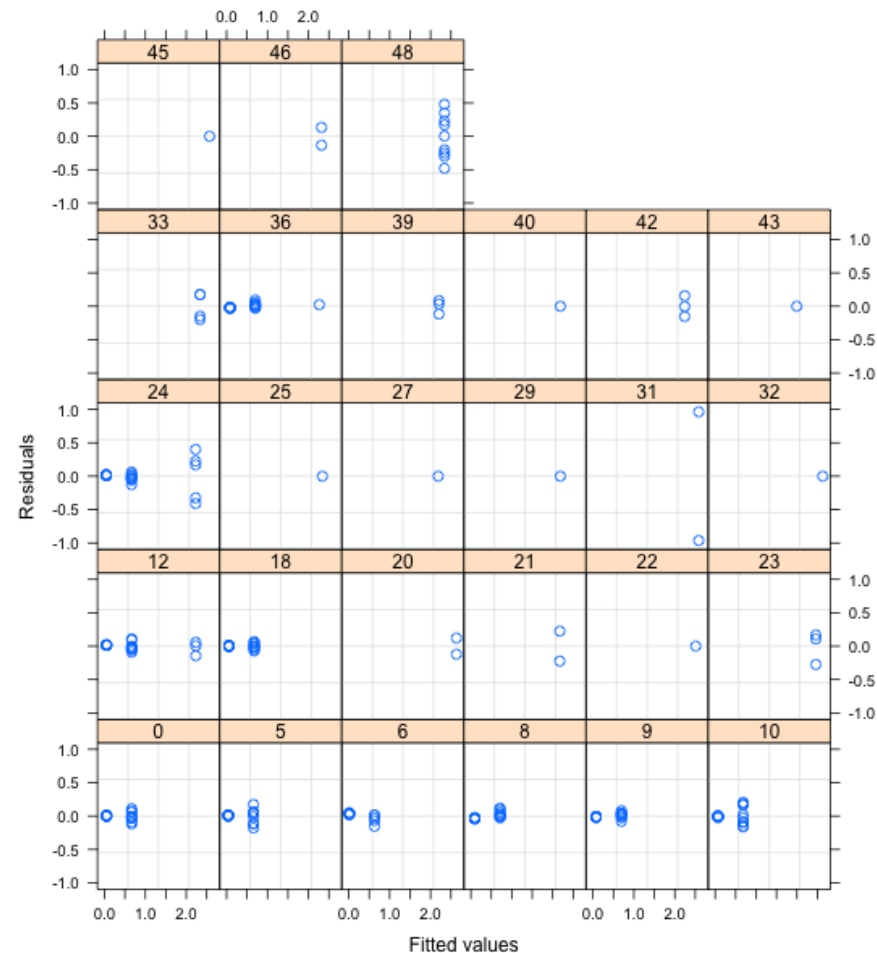
With parameter uncertainties



Black lines shows 99% quality level (posterior probability of CQA within specifications)

First prior was found poor to answer the question

- New data were available lately
 - stability data to assess uncertainty
 - 3 concentration levels (mg/mL)
 - 27 days of study
 - 3 runs per day
 - 3 repetitions per run
 - Sparse stability design
 - Many missing values
 - Interest in 2 variability sources
 - Residual variability
 - Effect of the runs
 - Mixed modeling

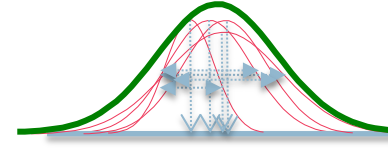
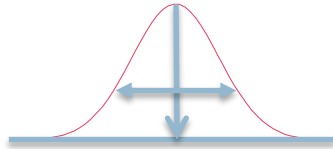


■ Model

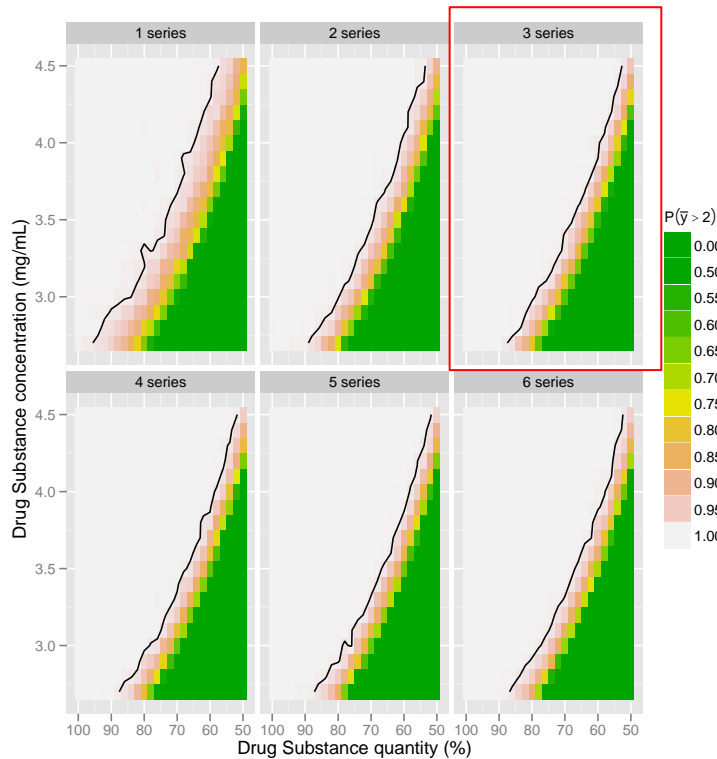
- $\text{Log-conc}_{ijkl} = \text{conc_target}_l + \text{time}_k + a_j + e_{ijkl}$
 - `conc_target` and `time` are used as qualitative factors
 - `a` is a random effect for the series: $a_j \sim N(\mu_a, \sigma_a)$
 - `e` is the residual error: $e_{ijkl} \sim N(0, \sigma_e)$

(fitted using the `MCMChregress` function of `MCMCpack` for R)

- Allows drawing new individual measurements, whatever the series
 - 1./ Sample a new series ($\tilde{a}^{(s)} \mid \text{data}$) from $N(\mu_a^{(s)} \mid \text{data}, \sigma_a \mid \text{data})$
 - 2./ Sample ($y^{(s)} \mid \text{data}$) from $N(\tilde{a}^{(s)} \mid \text{data}, \sigma_e^{(s)} \mid \text{data})$, $s=1 \dots \#\text{sampled elements}$
- Again, it is assumed that the computed total variability is appropriate for the range of concentration of interest

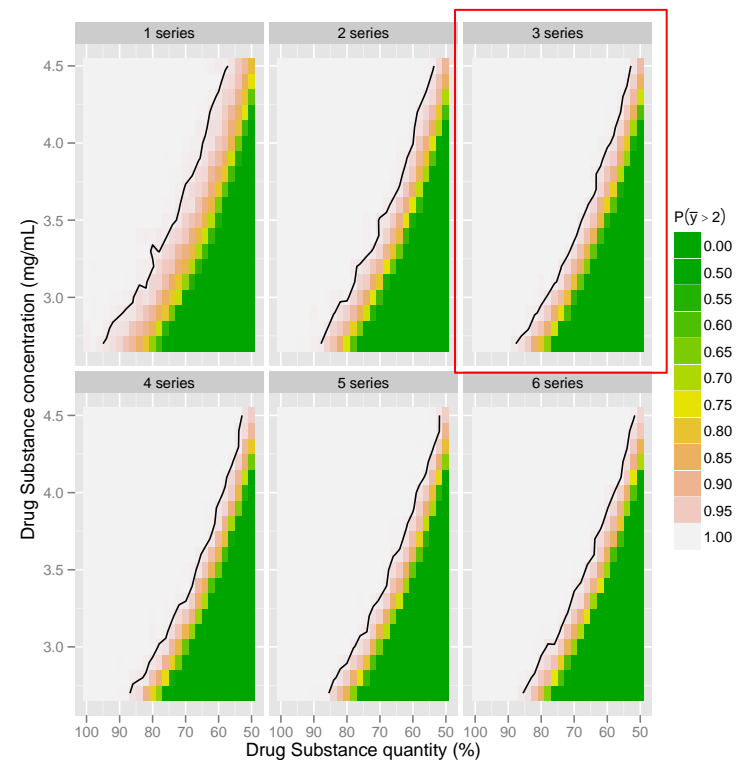


Without parameter uncertainties



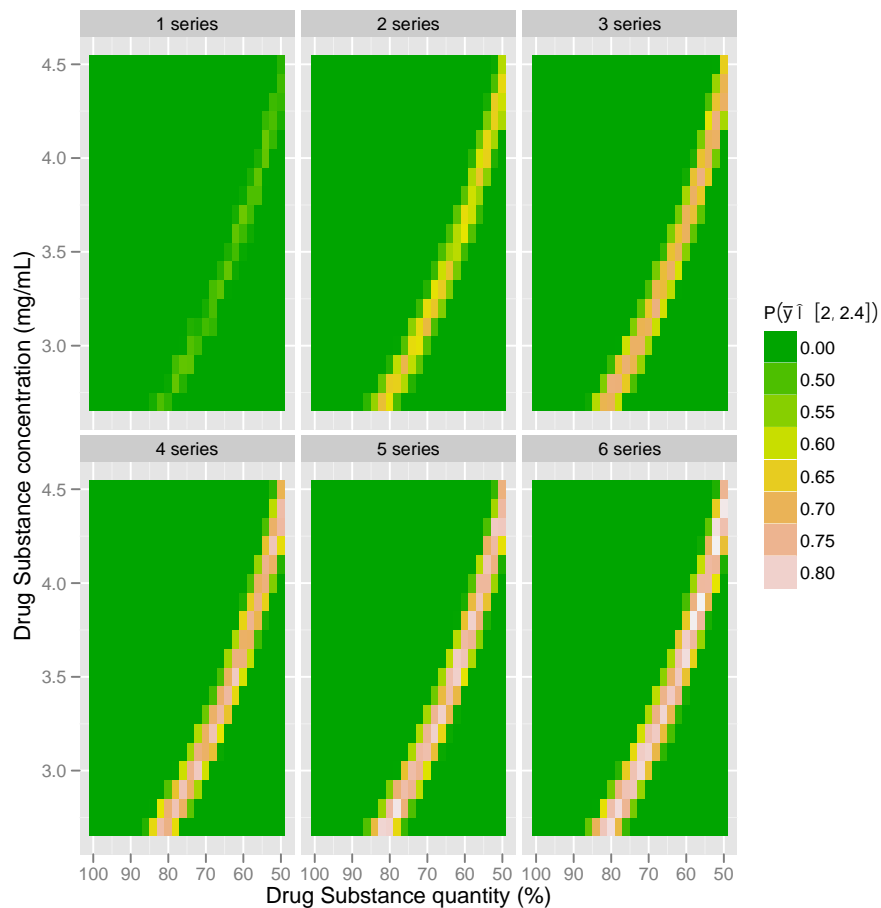
Safe dilution
easier to
achieve

With parameter uncertainties



This time, **no noticeable difference** between the simulations using **maximum likelihood estimators** and the ones using complete **posterior parameter uncertainties**
 → due to the higher amount of data, parameter uncertainties are very limited

- Secondary objective (reportable drug product result in [2, 2.4] mg/mL)



In this case, quality level never exceed 80%

Furthermore, the small “high quality” area gives low confidence in robustness against dilution error

Possibility to improve reportable result precision using 7, 8, 9, etc. series for both Bulk and Drug products, but costs increase as well

Now, the management can take a decision knowing the risks

- Effective Design Space is the tool to optimize a process/assay while concurrently assess its robustness
- Design Space allows providing guarantee that future runs will be on specifications
- It may be used even when available data are not perfect
 - To provide risk-based results
 - To allow efficient and knowingly decision
- The industrial impact of such an approach is important
 - → More companies are looking for this type of methodologies

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THANK YOU

- Arlenda is recruiting statisticians with knowledge in Bayesian statistics
 - In Europe in Brussel's area
 - In the USA, in Philadelphia area.

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