

# Utilizing Informative Priors in Group Comparisons with Completely Censored Data

# Introduction

**Censored data:** Common in the pharmaceutical industry

- e.g. differential expression of cytokines in the discovery space

**Approach:** Use Tobit or censored regression models

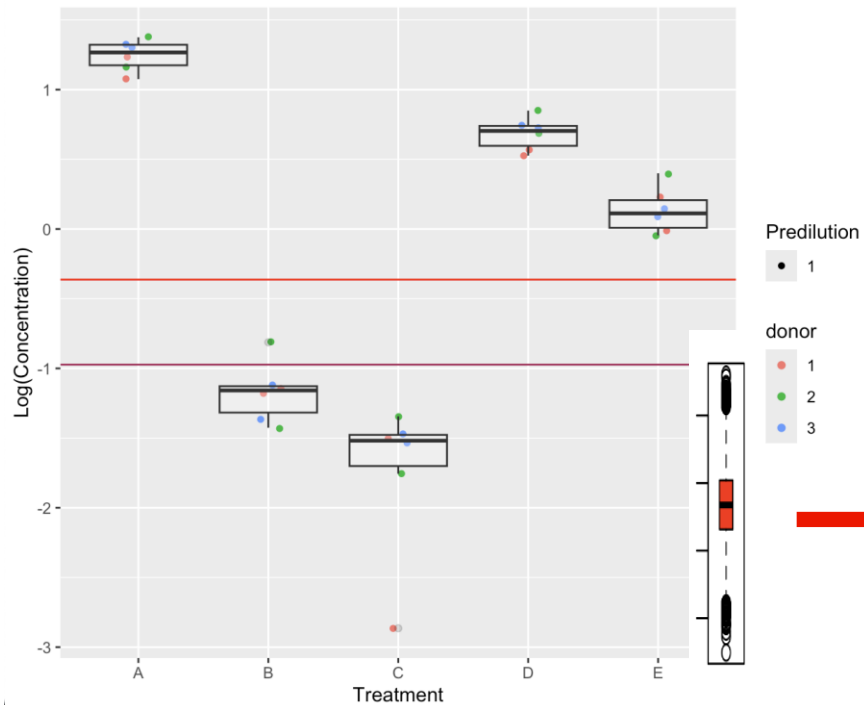
- **Assumption:**
  - The **Wald method** assumes that all group means are estimable.
  - **Problem:** Any complete censored group makes estimates unreliable.

**Potential solutions:**

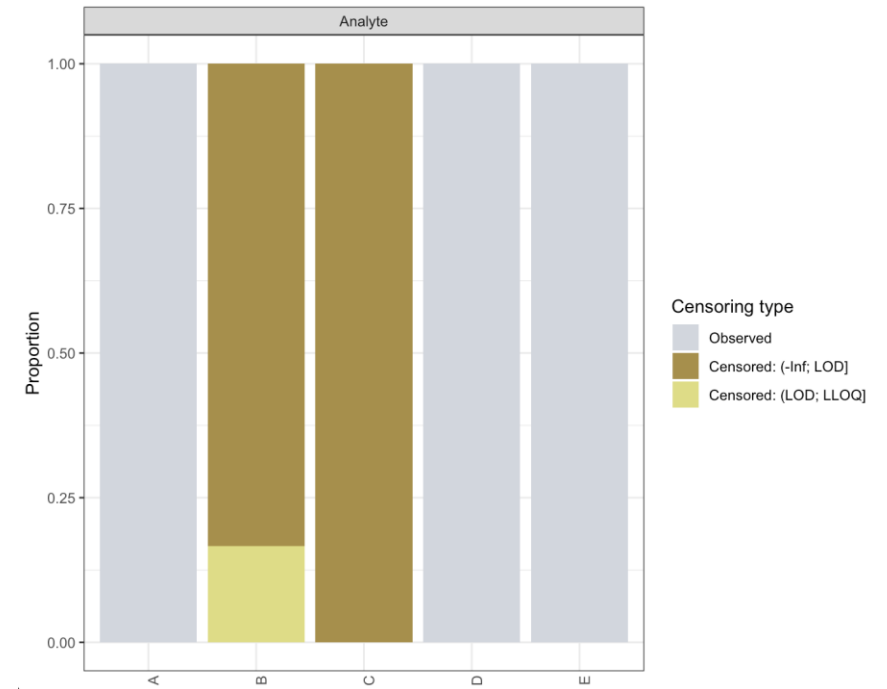
- **Profile Likelihood approach**
- **Bayesian Approach** with *informative priors*

# Data

- **Response:** Conc of Cytokine X
- **Treatments:** A, B, C, D and E
- **Detection Limits:**
  - **LOD** (*Limit of Detection*) : 0.378
  - **LLOQ** (*Lower Limit of Quantification*) : 0.6958



- **Censoring Challenges:**
  - **Types:** Left- and interval-censored values
  - **Critical issue:** Entirely censored treatment groups(2 groups)



# Bayesian analysis results

Non informative priors:

- An issue with Treatment C!

Treatment	Estimate	L95%CI	U95%CI
A	1.26	0.81	1.81
B	-1.13	-1.62	-0.58
C	$-6.32 \times 10^{21}$	$-1.91 \times 10^{22}$	$-1.95 \times 10^{20}$
D	0.70	0.23	1.26
E	0.14	-0.31	0.70

# Bayesian analysis results

Non informative priors:

- An issue with Treatment C!
- But why not Treatment B?
  - Mean=(log(LOD)+log(LLOQ))/2

Treatment	Estimate	L95%CI	U95%CI
A	1.26	0.81	1.81
B	-1.13	-1.62	-0.58
C	-6.32 X 10 <sup>21</sup>	-1.91 X 10 <sup>22</sup>	-1.95 X 10 <sup>20</sup>
D	0.70	0.23	1.26
E	0.14	-0.31	0.70

# Bayesian analysis results:

## Informative prior (Treatment C)

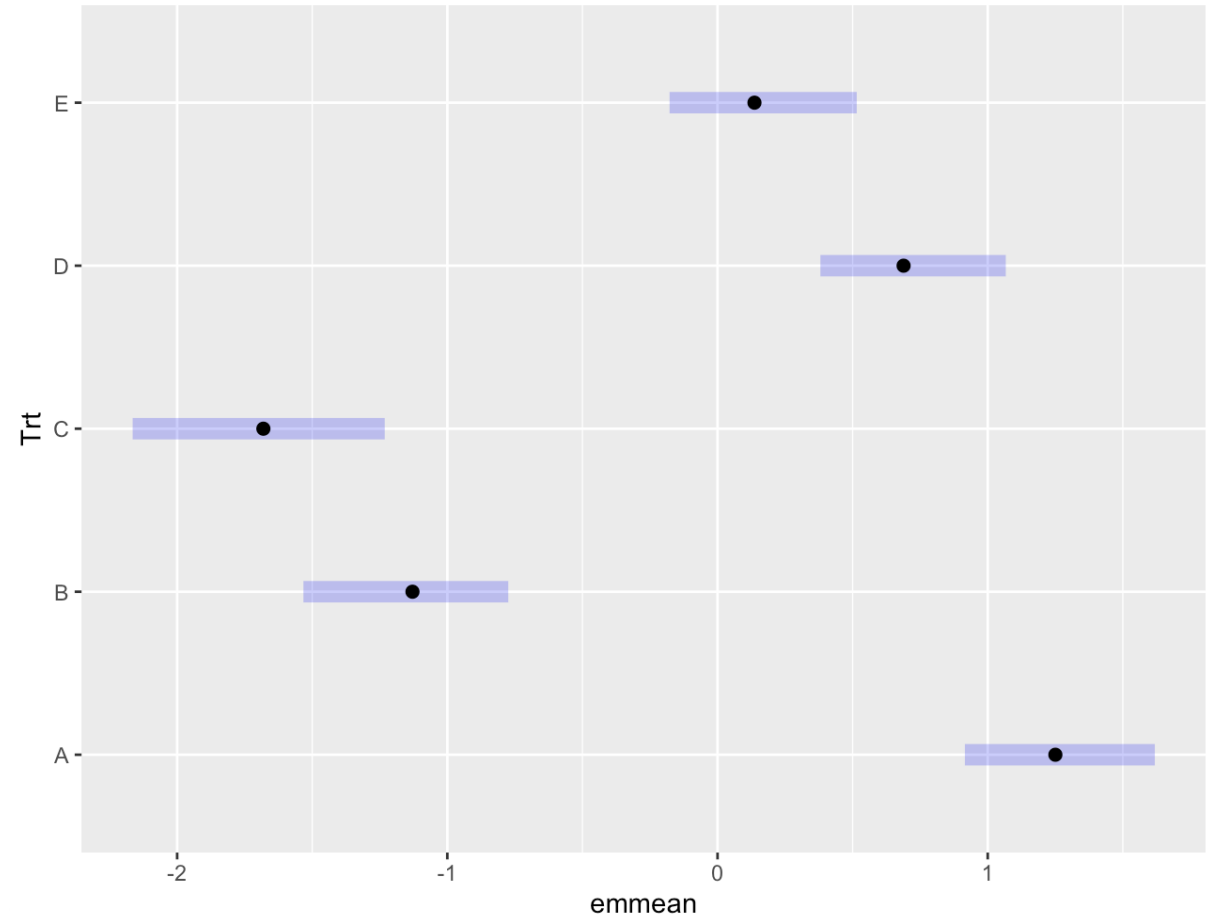
- Based on blank samples/control wells
  - $N(-1.66, 0.27^2)$
- Valid results

Treatment	Estimate	L95%CI	U95%CI
A	1.27	0.95	1.67
B	-1.12	-1.49	-0.71
C	-1.69	-2.18	-1.24
D	0.70	0.40	1.11
E	0.15	-0.16	0.55

# Bayesian analysis results

Treatment Group Comparison:

- **Treatment A:** Highest Cytokine X concentration
- **Note:** No difference between **Treatment B** and **Treatment C**



# Key Takeaways

## *What We've Learned*

- **Complete censored group:**
  - with only left-censored values lead to unreliable inference
  - but with some interval-censored values lead to valid inference
- **Bayesian approach:**
  - Allows for the incorporation of **informative priors** resulting in valid and robust analyses.

# References

1. Tobin, J. (1958). "Estimation of Relationships for Limited Dependent Variables." *Econometrica*, 26(1), 24-36.
2. Helsel, D. R. (2012). *Statistics for censored environmental data using Minitab and R* (2nd ed.). John Wiley & Sons.
3. Bürkner, P.-C. (2017). brms: An R package for Bayesian multilevel models using Stan. *Journal of Statistical Software*, 80(1), 1-28. <https://doi.org/10.18637/jss.v080.i01>

# Likelihood of censored data

$$L(\boldsymbol{\beta}, \sigma^2) = \prod_{i=1}^n \left[ \phi\left(\frac{Y_i - \mathbf{x}_i^T \boldsymbol{\beta}}{\sigma}\right)^{\delta_{i,0}} \left(1 - \Phi\left(\frac{R_i - \mathbf{x}_i^T \boldsymbol{\beta}}{\sigma}\right)\right)^{\delta_{i,R}} \left(\Phi\left(\frac{L_i - \mathbf{x}_i^T \boldsymbol{\beta}}{\sigma}\right)\right)^{\delta_{i,L}} \left(\Phi\left(\frac{R_i - \mathbf{x}_i^T \boldsymbol{\beta}}{\sigma}\right) - \Phi\left(\frac{L_i - \mathbf{x}_i^T \boldsymbol{\beta}}{\sigma}\right)\right)^{\delta_{i,I}} \right]$$

- $\boldsymbol{\beta}$  is the vector of regression coefficients.
- $\sigma^2$  is the variance of the normal distribution.
- $\mathbf{x}_i$  is the vector of covariates for observation  $i$ .
- $\mu_i = \mathbf{x}_i^T \boldsymbol{\beta}$  is the mean of the distribution for observation  $i$ .
- $\phi(\cdot)$  and  $\Phi(\cdot)$  are the PDF and CDF of the standard normal distribution, respectively.
- $\delta_{i,0}, \delta_{i,R}, \delta_{i,L}, \delta_{i,I}$  are indicator variables that are 1 if the observation  $i$  is exactly observed, right-censored, left-censored, or interval-censored, respectively, and 0 otherwise.