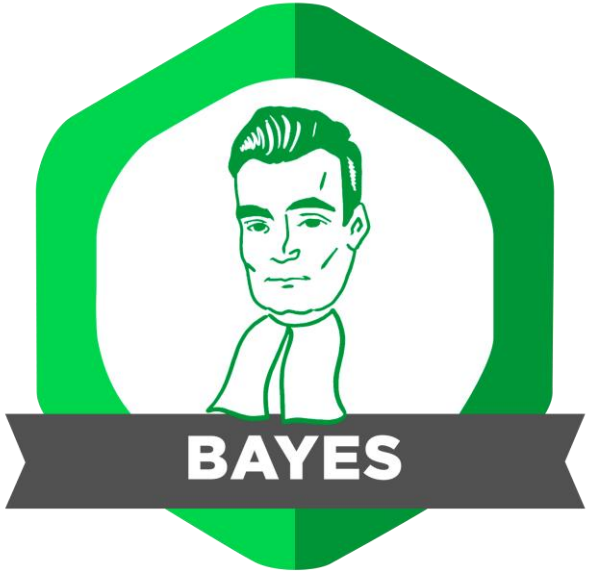




Flexible Bayesian Meta-Analysis with RoBMA R Package and JASP

František Bartoš

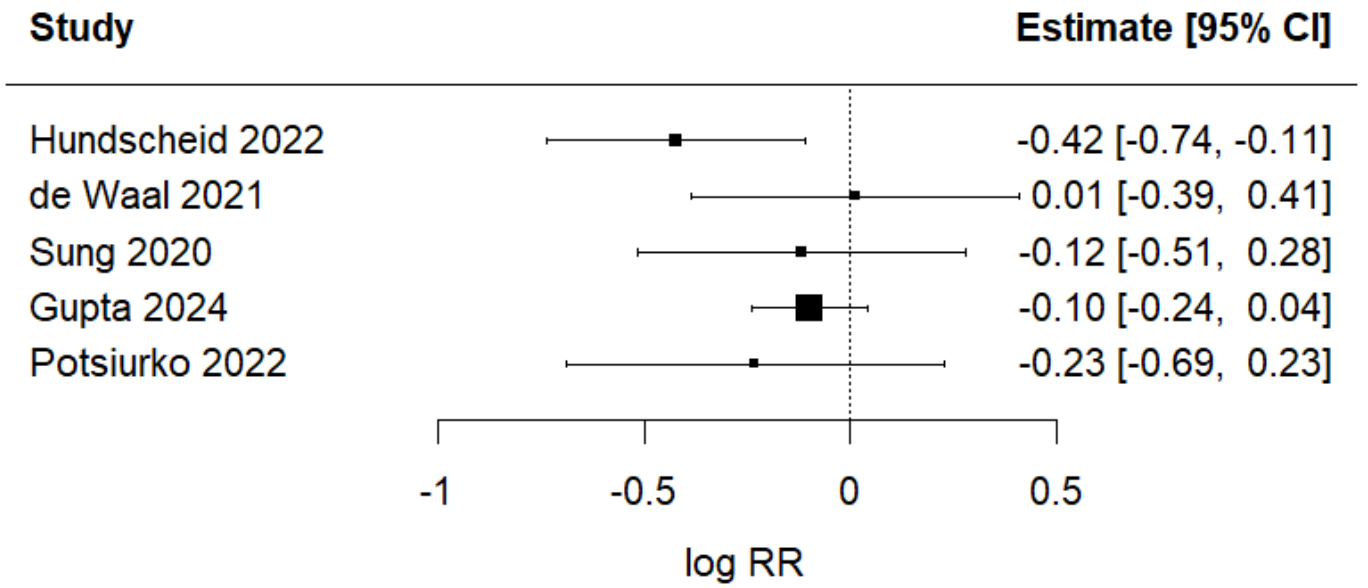
Why Bayesian Meta-Analysis



- Hypothesis tests are often underpowered
  => absence of evidence vs. evidence of absence
- All-or-none decisions based on statistical significance
  => uncertainty about the most appropriate model
- Poor convergence in small samples
  => parameter estimates (and CIs) might be undefined
- Effect size estimates conditional on heterogeneity estimates
  => parameter uncertainty is not propagated through the model (ad hoc corrections needed)
- Nonexistent sampling plan
  => accumulation bias, lack of validity of *p*-values for meta-analytic inference

Example 1: PDA in Newborns

- Patent ductus arteriosus (PDA) = congenital heart condition where a fetal blood vessel connecting the aorta and the pulmonary artery fails to close after birth
- Pharmacological treatment (Ibuprofen, paracetamol, …) vs placebo
- Outcomes
  - Mortality
  - Bronchopulmonary Dysplasia (chronic lung disease)



Villamor, E., Galán-Henríquez, G., Bartoš, F., & Gonzalez-Luis, G. E. (2025). Beneficial vs harmful effects of pharmacological treatment of patent ductus arteriosus: A Bayesian meta-analysis. *Pediatric Research*, *98*(2), 498-506.

Bayesian Meta-Analysis: Estimation

Data

- Effect size: y_i
- Standard error: se_i
- i = 1,2, …, K estimates

$$\mu \sim f()$$
$$\tau \sim g()$$
$$y_i \sim \text{Normal}(\mu, se_i^2 + \tau^2)$$

- Population mean effect μ and heterogeneity τ
- Prior distributions for all parameters

Prior Distributions

TABLE 6 Subfield-specific prior distributions for 46 individual topics from the Cochrane Database of Systematic Reviews estimated by hierarchical regression based on the complete data set

Topic	Comparisons	Studies	Prior δ	Prior τ
Acute Respiratory Infections	6	104	$\mathcal{T}(0, 0.38, 5)$	Inv-Gamma(1.73, 0.46)
Airways	46	815	$\mathcal{T}(0, 0.38, 6)$	Inv-Gamma(2.02, 0.28)
Anaesthesia	44	661	$\mathcal{T}(0, 0.55, 4)$	Inv-Gamma(1.62, 0.64)
Back and Neck	13	278	$\mathcal{T}(0, 0.37, 5)$	Inv-Gamma(1.75, 0.57)

- Empirical prior distribution based on Cochrane Database of Systematic Reviews
  - Overall and subfield specific
  - For SMD (Cohen's d), log OR, log RR, log HR, and OR

Bartoš, F., Gronau, Q. F., Timmers, B., Otte, W. M., Ly, A., & Wagenmakers, E. J. (2021). Bayesian model-averaged meta-analysis in medicine. *Statistics in Medicine*, *40*(30), 6743-6761.
Bartoš, F., Otte, W. M., Gronau, Q. F., Timmers, B., Ly, A., & Wagenmakers, E. J. (2023). Empirical prior distributions for Bayesian meta-analyses of binary and time to event outcomes. *arXiv Preprint*

Random-Effects Meta-Analysis (with `metafor`)

```
> df
             Study       logRR       sei
1 Hundscheid 2022 -0.4231647 0.1603412
2    de Waal 2021  0.0126000 0.2035837
3       Sung 2020 -0.1172685 0.2024090
4      Gupta 2024 -0.0980000 0.0717000
5  Potsiurko 2022 -0.2318016 0.2338914


> rma(yi = df$logRR, sei = df$sei)
…
Test for Heterogeneity:
Q(df = 4) = 4.1902, p-val = 0.3809

Model Results:

estimate      se     zval    pval    ci.lb    ci.ub
 -0.1531  0.0702  -2.1819  0.0291  -0.2906  -0.0156  *
```

Random-Effects Meta-Analysis (with RoBMA)

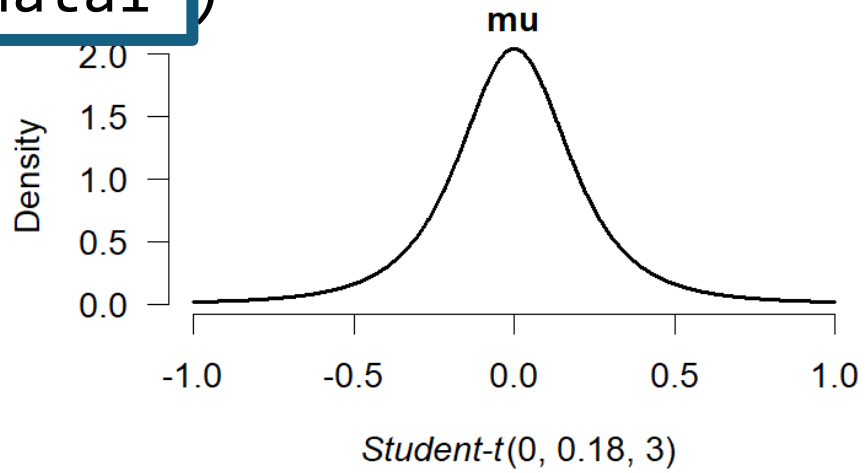
```
fit.brma <- brma(yi = df$logRR, sei = df$sei,
                 measure = "RR",
                 prior_informed_field = "medicine",
                 prior_informed_subfield = "neonatal")
```

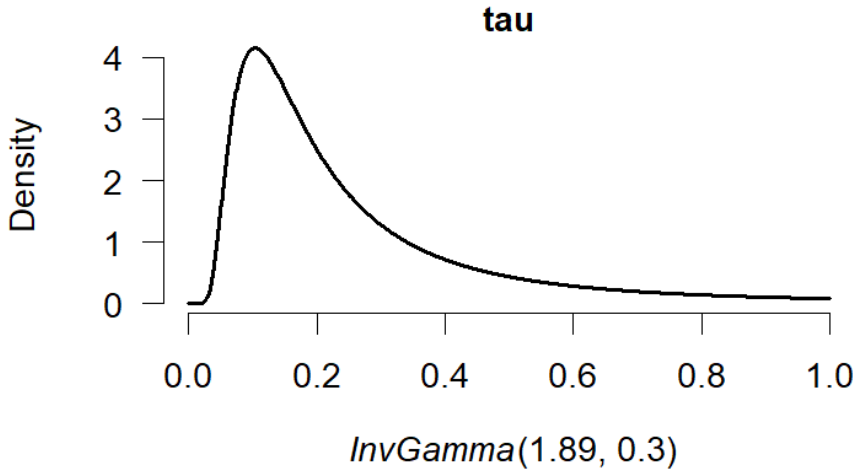
```
> print_prior(fit.brma)
mu:   Student-t(0, 0.18, 3)
tau:  InvGamma(1.89, 0.3)

> summary(fit.brma)

Bayesian Random-Effects Model (k = 5)

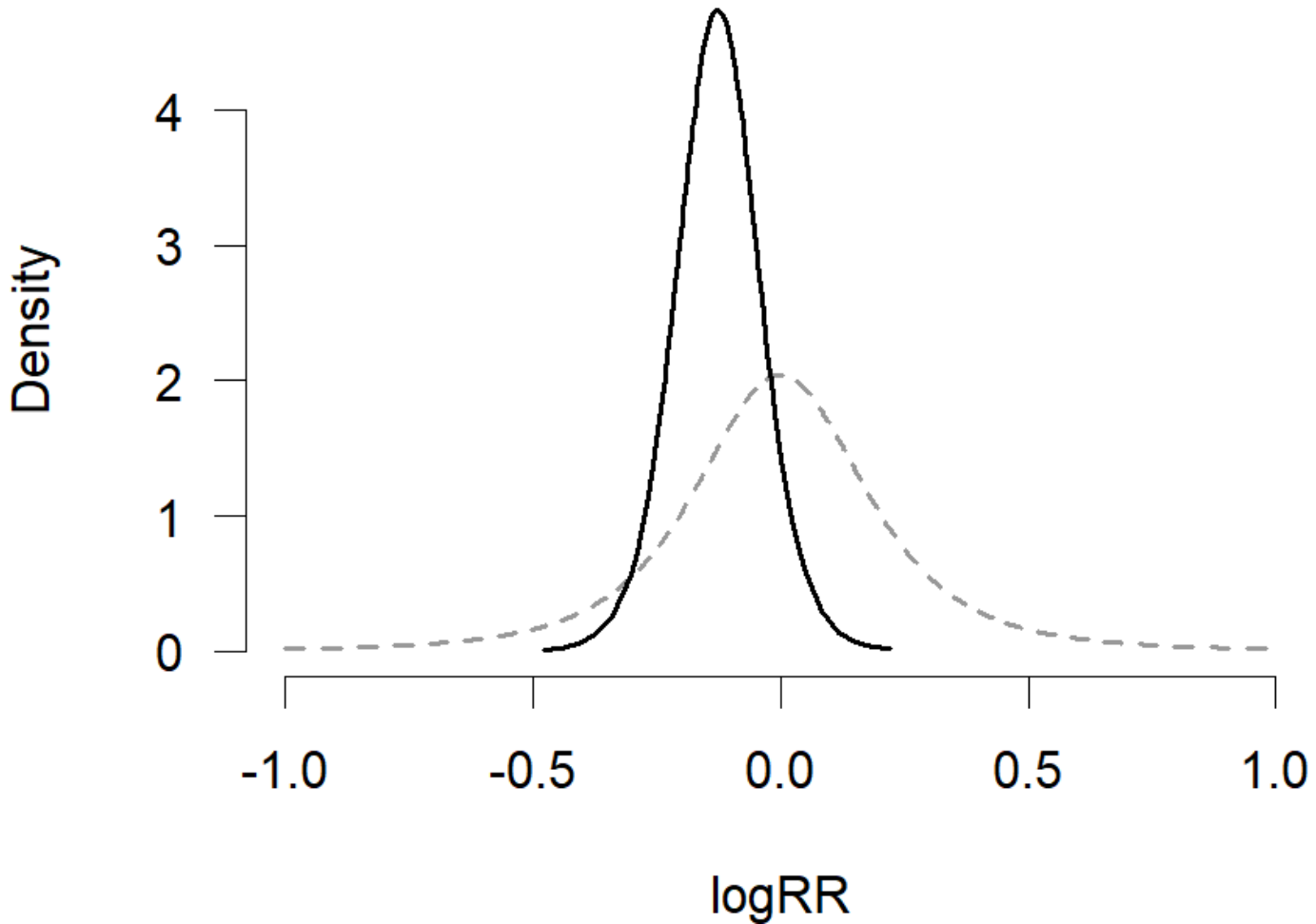
Estimates
      Mean    SD  0.025    0.5 0.975
mu  -0.127 0.089 -0.301 -0.127 0.049
tau  0.144 0.084  0.049  0.124 0.359
```





Prior and Postrerior Distribution

```
> plot(fit.brma, parameter = "mu", prior = TRUE, …)
```



Bayesian Meta-Analysis: Hypothesis Testing

- Test for the presence vs. absence of the effect (in the random-effects model)

=> comparing prior predictive performance of

- H_0: random-effects model assuming absence of the effect
- H_1: random-effects model assuming presence of the effect

$$\mathrm{BF}_{10} = \frac{p(\mathrm{data} \mid \mathcal{H}_1)}{p(\mathrm{data} \mid \mathcal{H}_0)}$$

H_0 :

$$\mu = 0$$
$$\tau \sim g()$$
$$\mathrm{y}_i \sim \mathrm{Normal}(\mu, \mathrm{se}_i^2 + \tau^2)$$

H_1 :

$$\mu \sim f()$$
$$\tau \sim g()$$
$$\mathrm{y}_i \sim \mathrm{Normal}(\mu, \mathrm{se}_i^2 + \tau^2)$$

Bayesian Meta-Analysis: Hypothesis Testing

- Test for the presence vs. absence of heterogeneity (in the random-effects model)

=> comparing prior predictive performance of

- H_f: random-effects model assuming absence of heterogeneity
- H_r: random-effects model assuming presence of heterogeneity

$$\mathrm{BF}_{\mathrm{rf}} = \frac{p(\mathrm{data} \mid \mathcal{H}_{\mathrm{r}})}{p(\mathrm{data} \mid \mathcal{H}_{\mathrm{f}})}$$

H_f :

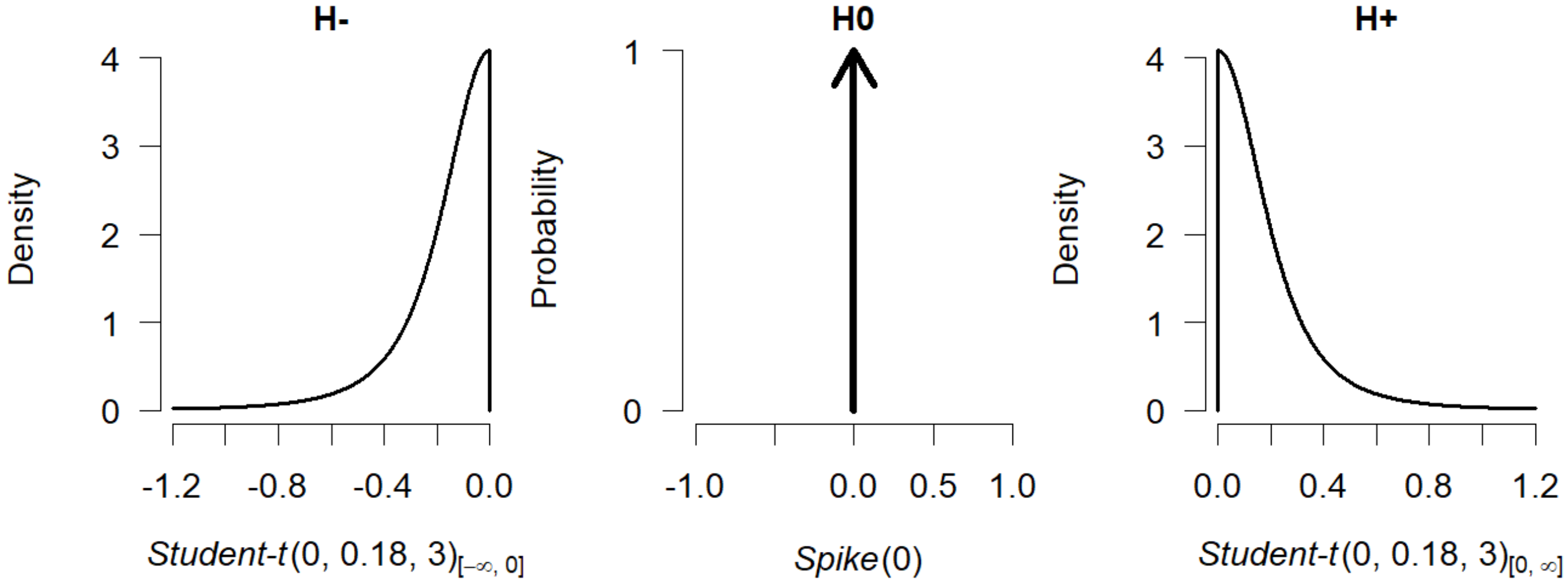
$$\mu \sim f()$$
$$\tau = 0$$
$$\mathrm{y}_i \sim \mathrm{Normal}(\mu, \mathrm{se}_i^2 + \tau^2)$$

H_{1r} :

$$\mu \sim f()$$
$$\tau \sim g()$$
$$\mathrm{y}_i \sim \mathrm{Normal}(\mu, \mathrm{se}_i^2 + \tau^2)$$

Informed Prior Distributions

```
prior_minus <- prior("Student-t", list(0, 0.18, 3), list(-Inf, 0))
prior_plus  <- prior("Student-t", list(0, 0.18, 3), list(0, Inf))
prior_null  <- prior("Spike", list(0))
```



Informed Hypothesis Tests

```
fit.brma_minus <- brma(yi = df$logRR, sei = df$sei, measure = "RR",
          prior_effect = prior_minus, prior_heterogeneity = prior_het)

fit.brma_plus  <- brma(yi = df$logRR, sei = df$sei, measure = "RR",
          prior_effect = prior_plus, prior_heterogeneity = prior_het)

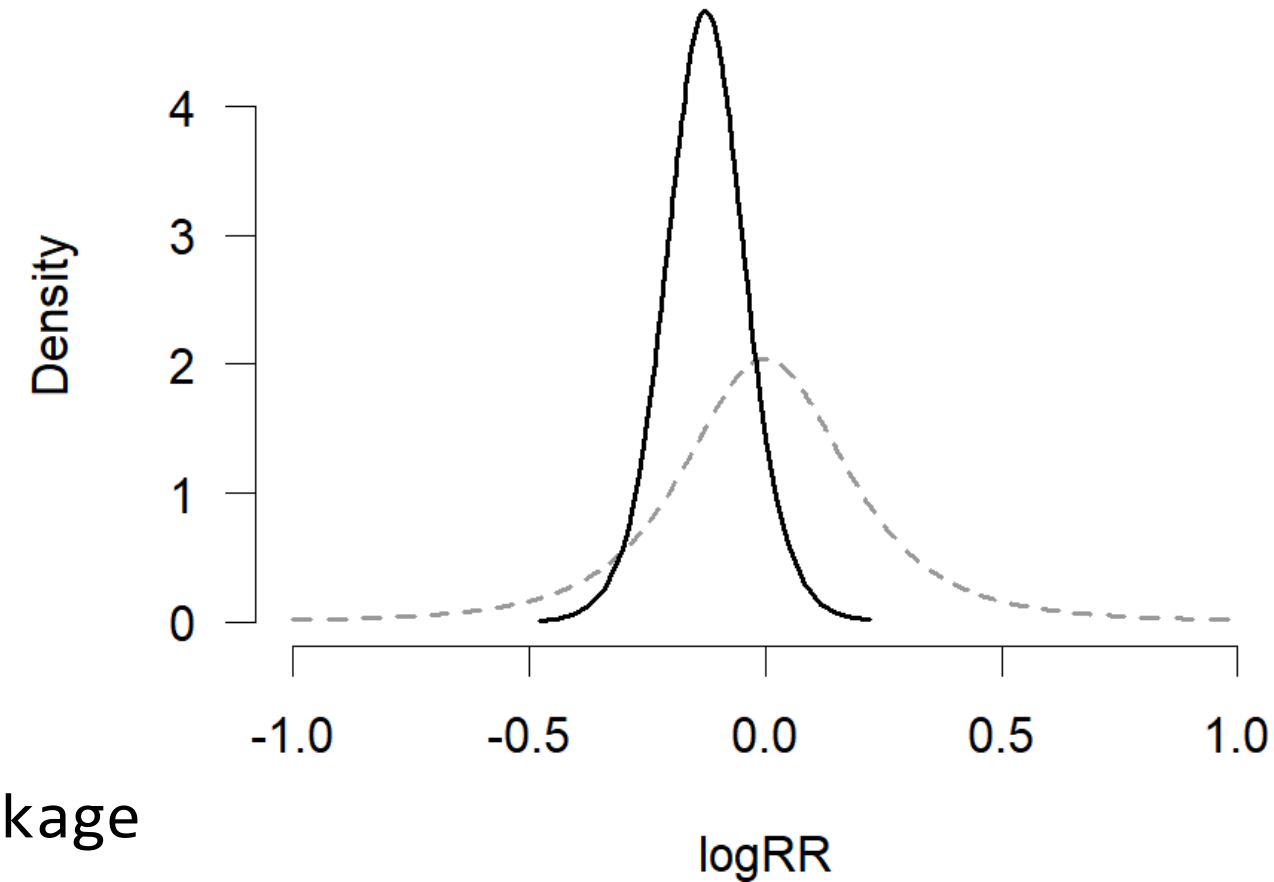
fit.brma_null  <- brma(yi = df$logRR, sei = df$sei, measure = "RR",
          prior_effect = prior_null, prior_heterogeneity = prior_het)

fit.brma       <- add_marglik(fit.brma)
…
fit.brma_null  <- add_marglik(fit.brma_null)

> bf(fit.brma, fit.brma_null)
Bayes factor in favor of Model 1 over Model 2: 1.40731
> bf(fit.brma_minus, fit.brma_null)
Bayes factor in favor of Model 1 over Model 2: 2.60695
> bf(fit.brma_minus, fit.brma_plus)
Bayes factor in favor of Model 1 over Model 2: 12.70025
```

Informed Hypothesis Tests from Density*

```
> hypothesis(fit.brma, c(
    "mu != 0 vs mu = 0",
    "mu  < 0 vs mu = 0",
    "mu  < 0 vs mu > 0"))
 Alternative:     Null:         BF
      mu != 0    mu = 0     1.400
       mu < 0    mu = 0     2.604
       mu < 0    mu > 0    13.245
```



*in the upcoming version of the package

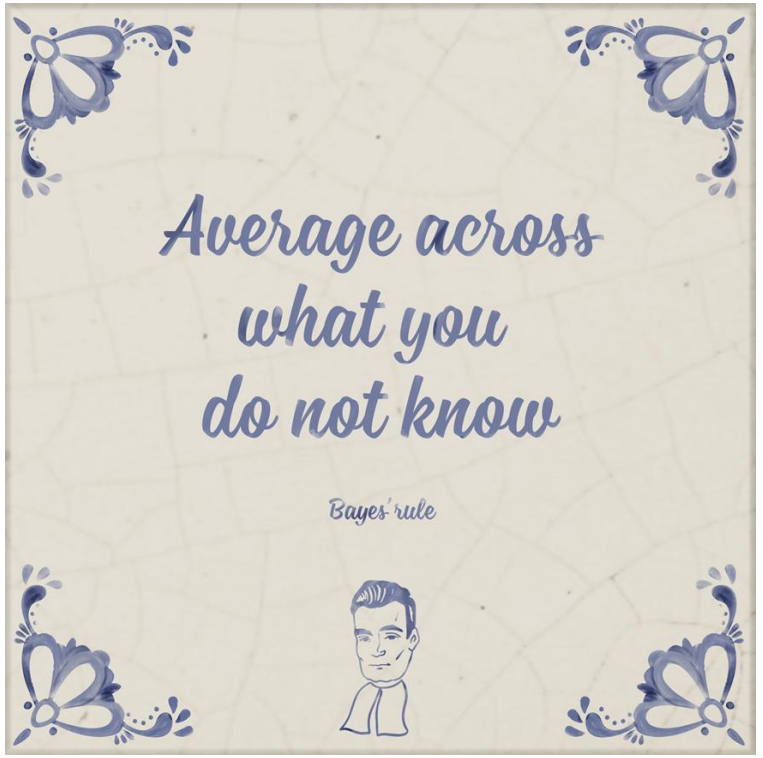
Bayesian Model-Averaging

- The true model is unknown

=> combine models according to their predictive performance

=> propagate uncertainty across models





Bayesian Model-Averaging

- Combining assumptions about the effect and heterogeneity:

1. the fixed-effect null hypothesis $\mathcal{H}_0^f : \mu = 0 \ , \tau = 0$
2. the fixed-effect alternative hypothesis $\mathcal{H}_1^f : \mu \sim f() \ , \tau = 0$
3. the random-effects null hypothesis $\mathcal{H}_0^r : \mu = 0, \tau \sim g()$
4. the random-effects alternative hypothesis $\mathcal{H}_1^r : \mu \sim f() \ , \tau \sim g()$

- Model uncertainty incorporated via prior model probabilities

=> more priors to be specified (prior model probabilities)

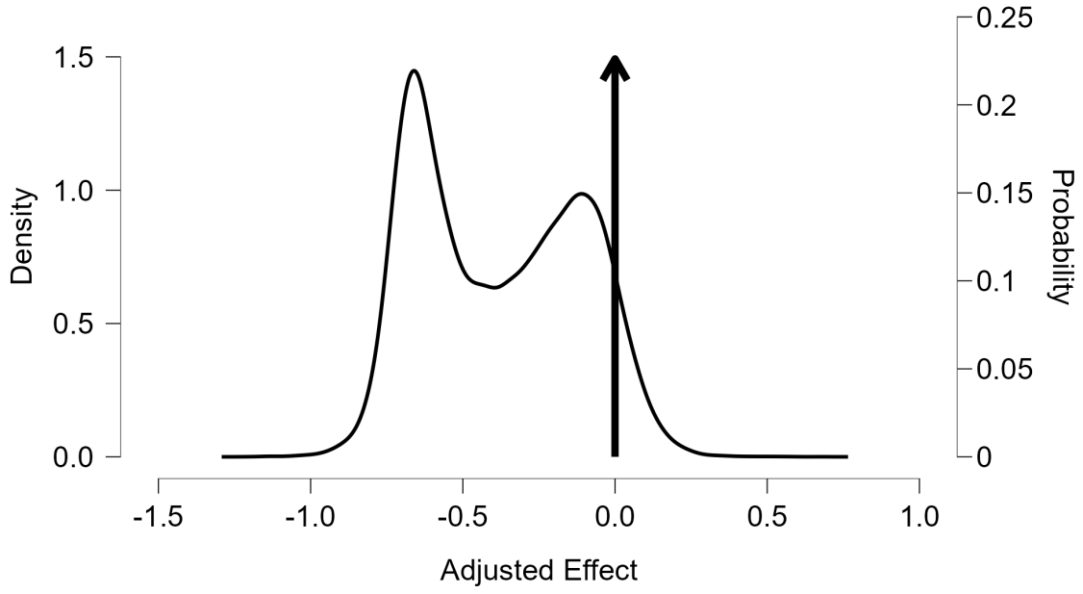
- But single model inference equals to setting the prior model probability to 1

Bayesian Model-Averaged Estimates

- Model-averaged posterior distribution
  - Combines estimates from each model proportionately to the posterior model probability

$$p(\theta|\text{data}) = \sum_{n=1}^{N} p(\theta_n|\mathcal{H}_n, \text{data})\, p(\mathcal{H}_n|\text{data})$$

  - Regularized to zero if models assuming absence of the effect have posterior probability

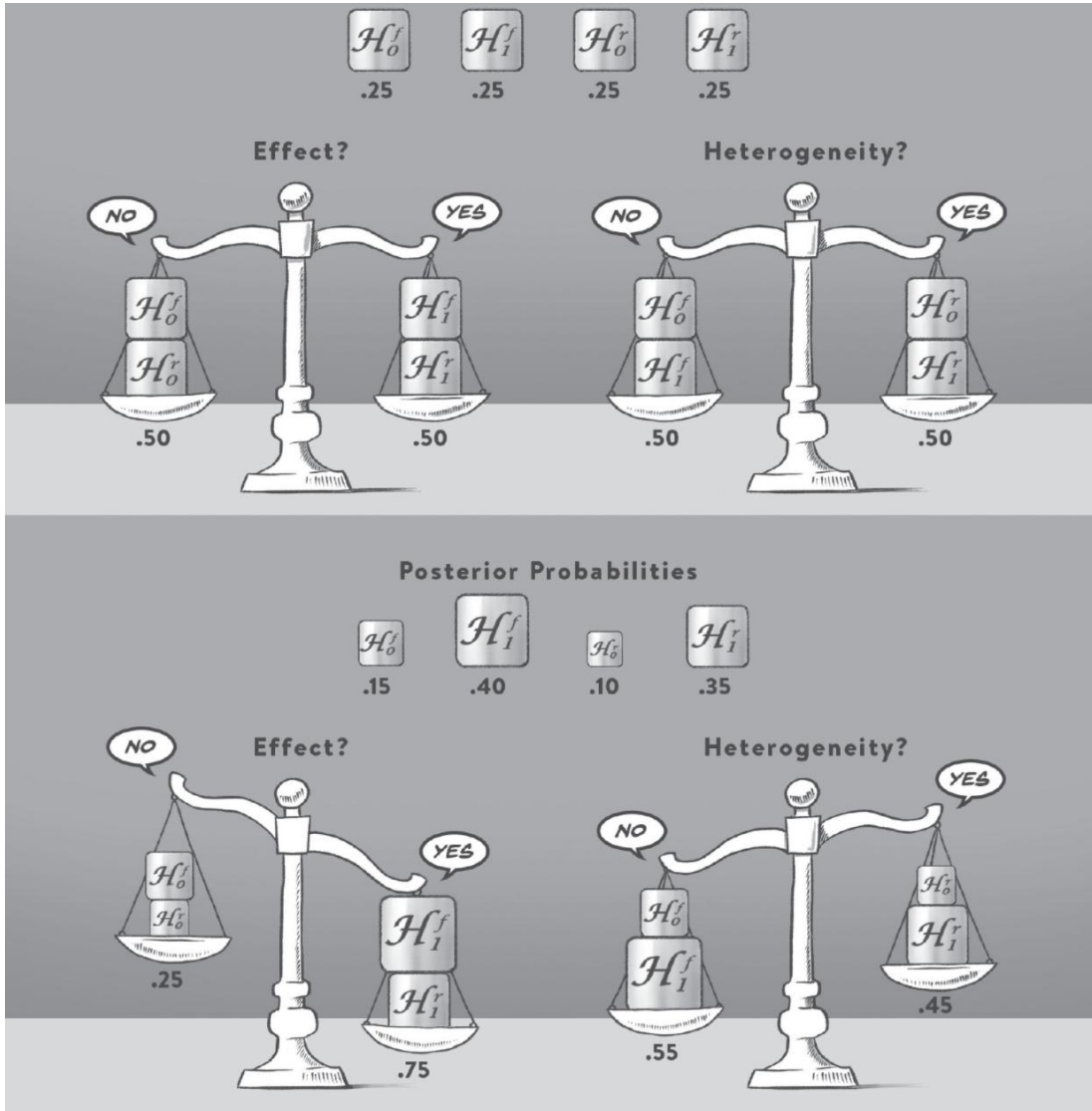


Bayesian Model-Averaged Hypothesis Tests

- Inclusion Bayes factors:

$$\underbrace{\mathrm{BF}_{10}}_{\substack{\text{Inclusion BF}\\ \text{for effect}}} = \underbrace{\frac{p(\mathscr{H}_1^f \mid \text{data}) + p(\mathscr{H}_1^r \mid \text{data})}{p(\mathscr{H}_0^f \mid \text{data}) + p(\mathscr{H}_0^r \mid \text{data})}}_{\substack{\text{Posterior inclusion odds}\\ \text{for effect}}} \Big/ \underbrace{\frac{p(\mathscr{H}_1^f) + p(\mathscr{H}_1^r)}{p(\mathscr{H}_0^f) + p(\mathscr{H}_0^r)}}_{\substack{\text{Prior inclusion odds}\\ \text{for effect}}}$$

$$\underbrace{\mathrm{BF}_{rf}}_{\substack{\text{Inclusion BF}\\ \text{for heterogeneity}}} = \underbrace{\frac{p(\mathscr{H}_0^r \mid \text{data}) + p(\mathscr{H}_1^r \mid \text{data})}{p(\mathscr{H}_0^f \mid \text{data}) + p(\mathscr{H}_1^f \mid \text{data})}}_{\substack{\text{Posterior inclusion odds}\\ \text{for heterogeneity}}} \Big/ \underbrace{\frac{p(\mathscr{H}_0^r) + p(\mathscr{H}_1^r)}{p(\mathscr{H}_0^f) + p(\mathscr{H}_1^f)}}_{\substack{\text{Prior inclusion odds}\\ \text{for heterogeneity}}}$$



Gronau, Q. F., Heck, D. W., Berkhout, S. W., Haaf, J. M., & Wagenmakers, E. J. (2021). A primer on Bayesian model-averaged meta-analysis. *Advances in Methods and Practices in Psychological Science*, *4*(3)

Bayesian Model Averaging

```
fit.BMA <- BMA(yi = df$logRR, sei = df$sei,
               measure = "RR",
               prior_informed_field = "medicine",
               prior_informed_subfield = "neonatal")

> summary(fit.BMA)

Bayesian Model-Averaged Random-Effects Model (k = 5)

Component Inclusion
                Prior prob. Post. prob. Inclusion BF
Effect               0.500       0.714         2.498
Heterogeneity        0.500       0.416         0.712

Estimates
      Mean    SD  0.025    0.5 0.975
mu  -0.091 0.082 -0.252 -0.093 0.000
tau  0.064 0.093  0.000  0.000 0.293
```

Model Summary

```
fit.BMA <- BMA(yi = df$logRR, sei = df$sei,
               measure = "RR",
               prior_informed_field = "medicine",
               prior_informed_subfield = "neonatal")

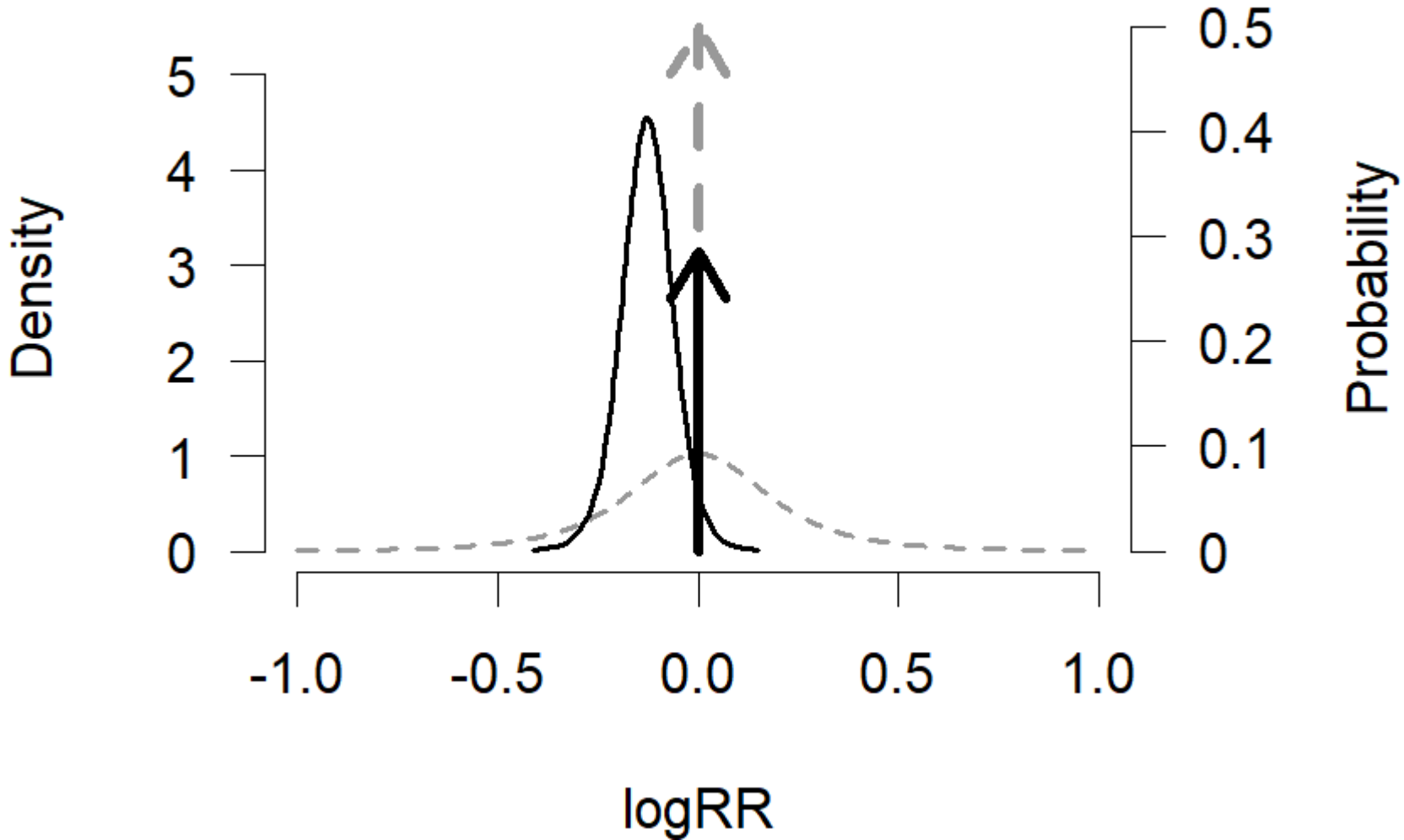
> summary_models(fit.BMA, type = "individual")

Individual Models
```

Effect	Heterogeneity	Prior prob.	Post. prob.	Inclusion BF
Spike(0)	Spike(0)	0.250	0.116	0.392
Student-t(0, 0.18, 3)	Spike(0)	0.250	0.469	2.645
Spike(0)	InvGamma(1.89, 0.3)	0.250	0.170	0.616
Student-t(0, 0.18, 3)	InvGamma(1.89, 0.3)	0.250	0.246	0.976

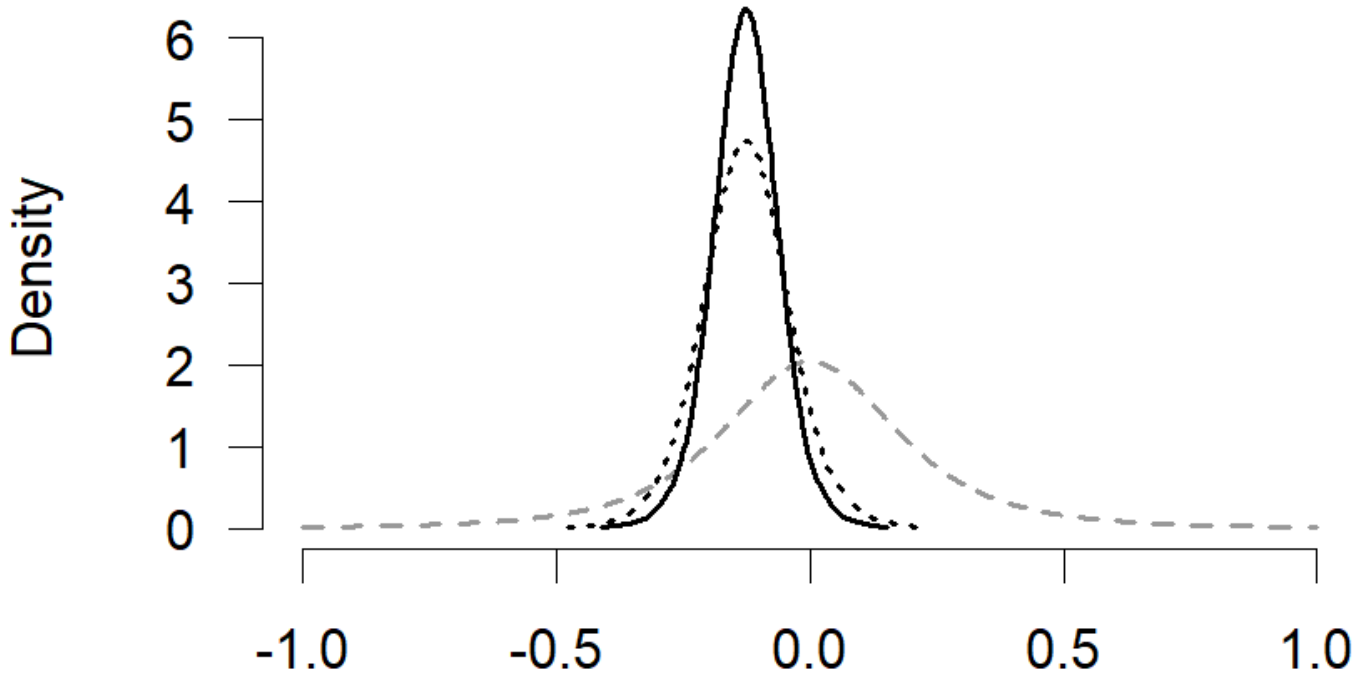
Model-Averaged Prior and Posterior Distribution

```
> plot(fit.BMA, parameter = "mu", prior = TRUE, …)
```

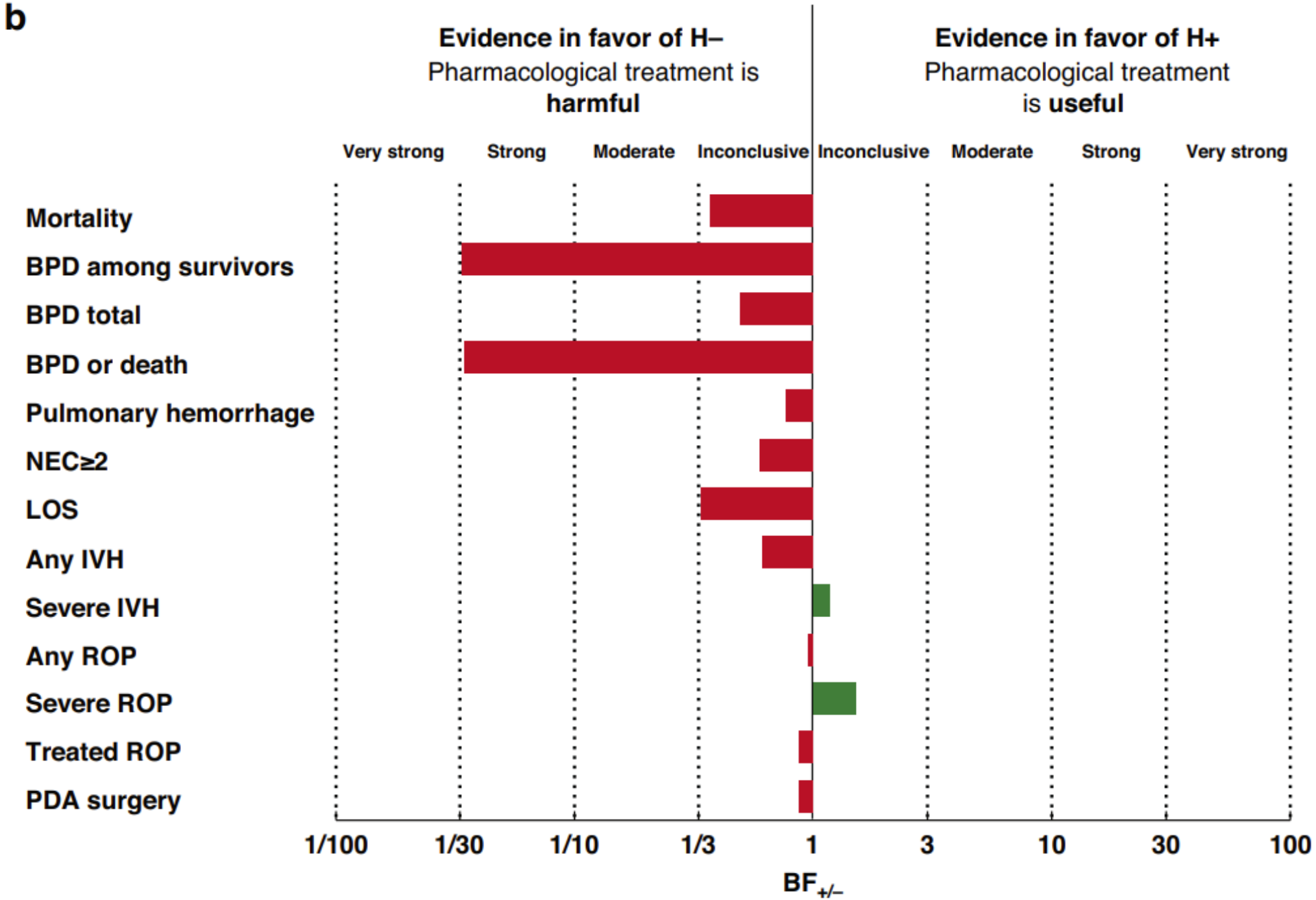


Model-Averaged Informed Hypothesis Tests from Density*

```
> hypothesis(fit.BMA, c(
    "mu != 0 vs mu = 0",
    "mu  < 0 vs mu = 0",
    "mu  < 0 vs mu > 0"), conditional = TRUE)
 Alternative:     Null:         BF
       mu != 0    mu = 0     2.438
        mu < 0    mu = 0     4.730
        mu < 0    mu > 0    32.267
```



*in the upcoming version of the package


b
Evidence in favor of H–
Pharmacological treatment is
harmful
Evidence in favor of H+
Pharmacological treatment
is useful
Very strong
Strong
Moderate
Inconclusive
Inconclusive
Moderate
Strong
Very strong
Mortality
BPD among survivors
BPD total
BPD or death
Pulmonary hemorrhage
NEC≥2
LOS
Any IVH
Severe IVH
Any ROP
Severe ROP
Treated ROP
PDA surgery
1/100
1/30
1/10
1/3
1
3
10
30
100
BF+/–

Publication Bias

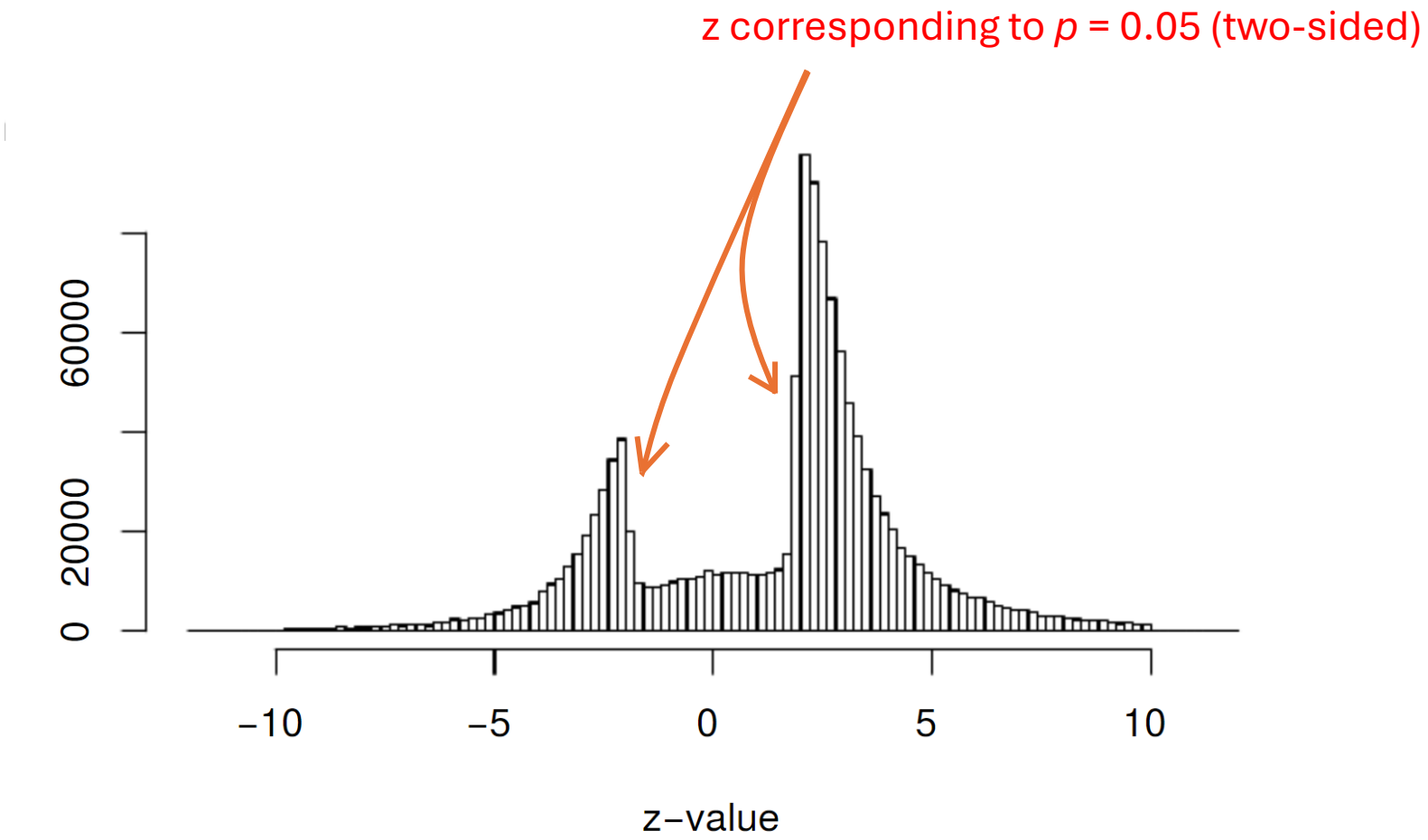


FIGURE 1 The distribution of more than one million *z*-values from Medline (1976–2019).

van Zwet, E. W., & Cator, E. A. (2021). The significance filter, the winner's curse and the need to shrink. *Statistica Neerlandica*

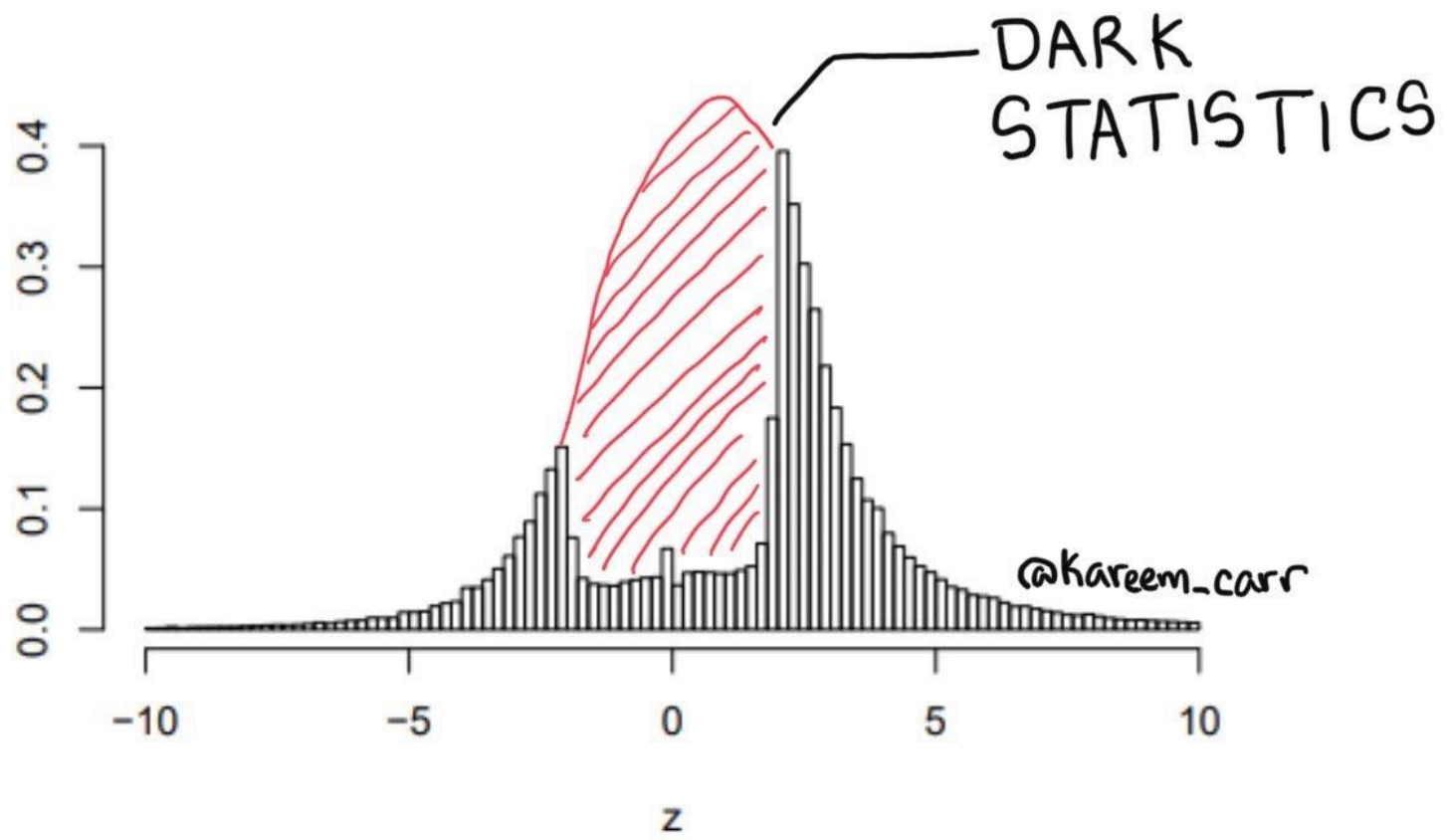


Figure 1: The distribution of more than one million z-values from Medline (1976–2019).

https://twitter.com/kareem_carr/status/1335282974443655168/photo

Publication Bias Adjustment Methods

- Models adjusting for relationship between effect sizes and standard errors
  - Trim and fill (Duval & Tweedie, 2000)
  - PET-PEESE (Stanley & Doucouliagos, 2014)
  - EK (Bom & Rachinger, 2019)
- Selection models of *p*-values
  - 3PSM, 4PSM (Vevea & Hedges, 1995)
  - AK1, AK2 (Andrews & Kasy, 2019)
  - *p*-curve (Simonsohn et al., 2014)
  - *p*-uniform (Van Assen et al., 2015)

Robust Bayesian Meta-Analysis

```
fit.RoBMA <- RoBMA(yi = df$logRR, sei = df$sei,
                   measure = "RR",
                   prior_informed_field = "medicine",
                   prior_informed_subfield = "neonatal")

> summary(fit.RoBMA)

Robust Bayesian Model-Averaged Random-Effects Model (k = 5)

Component Inclusion
                 Prior prob. Post. prob. Inclusion BF
Effect                 0.500       0.564        1.296
Heterogeneity          0.500       0.386        0.628
Publication Bias       0.500       0.548        1.210

Estimates
      Mean    SD  0.025    0.5 0.975
mu  -0.059 0.079 -0.230 -0.013 0.040
tau  0.057 0.088  0.000  0.000 0.280
```

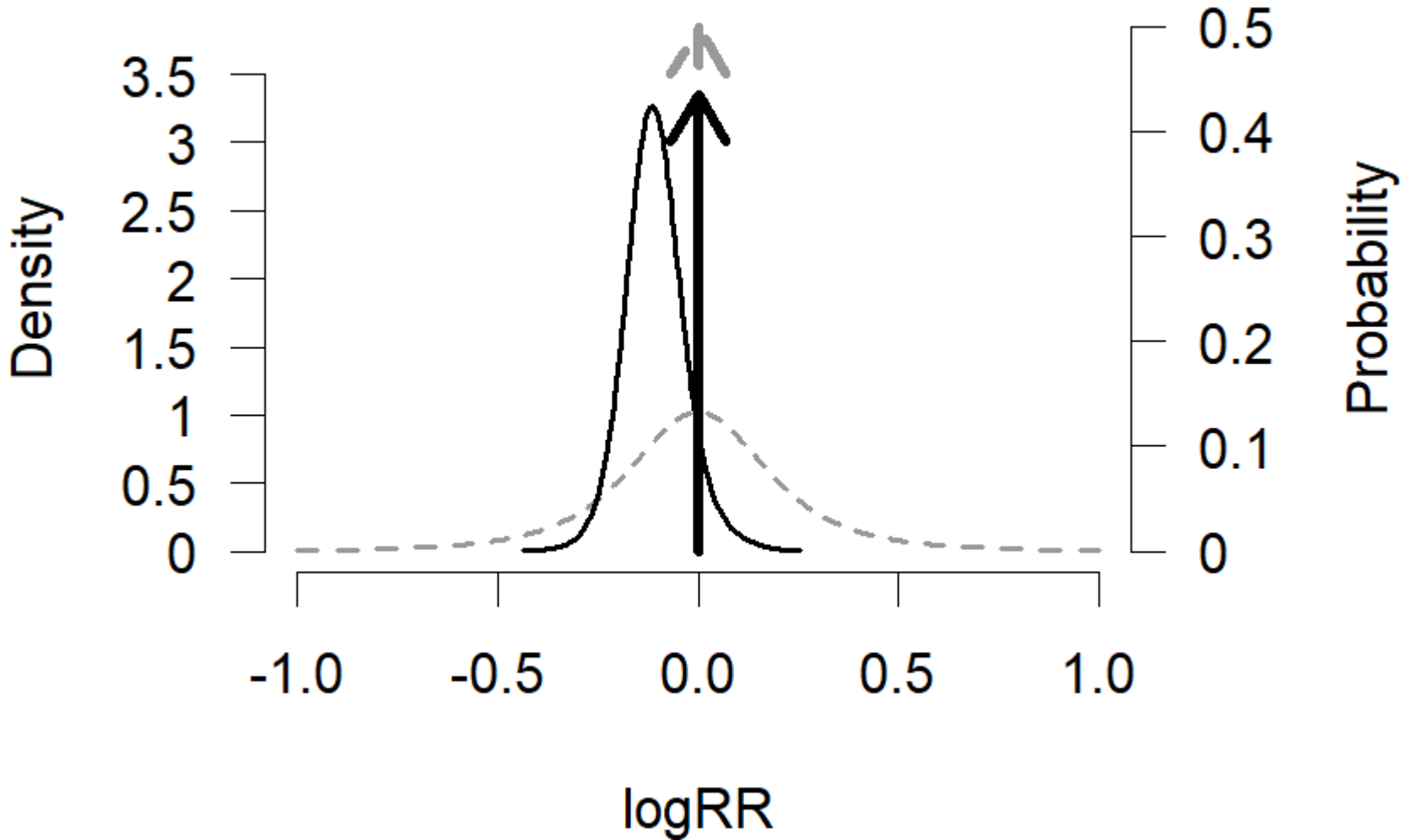
Model Summary

```
fit.RoBMA <- RoBMA(yi = df$logRR, sei = df$sei,
                   measure = "RR",
                   prior_informed_field = "medicine",
                   prior_informed_subfield = "neonatal")
```

```
> summary_models(fit.RoBMA)
…
Publication Bias
                                 Hypothesis  Prior prob.  Post. prob.  Inclusion BF
None                             Null              0.500        0.452         0.826
omega[two-sided: .05]            Alternative       0.042        0.030         0.700
omega[two-sided: .05, .1]        Alternative       0.042        0.020         0.473
omega[one-sided: .05]            Alternative       0.042        0.030         0.702
omega[one-sided: .025, .05]      Alternative       0.042        0.027         0.645
omega[one-sided: .05, .5]        Alternative       0.042        0.040         0.958
omega[one-sided: .025, .05, .5] Alternative       0.042        0.039         0.932
PET                              Alternative       0.125        0.215         1.912
PEESE                            Alternative       0.125        0.148         1.212
```

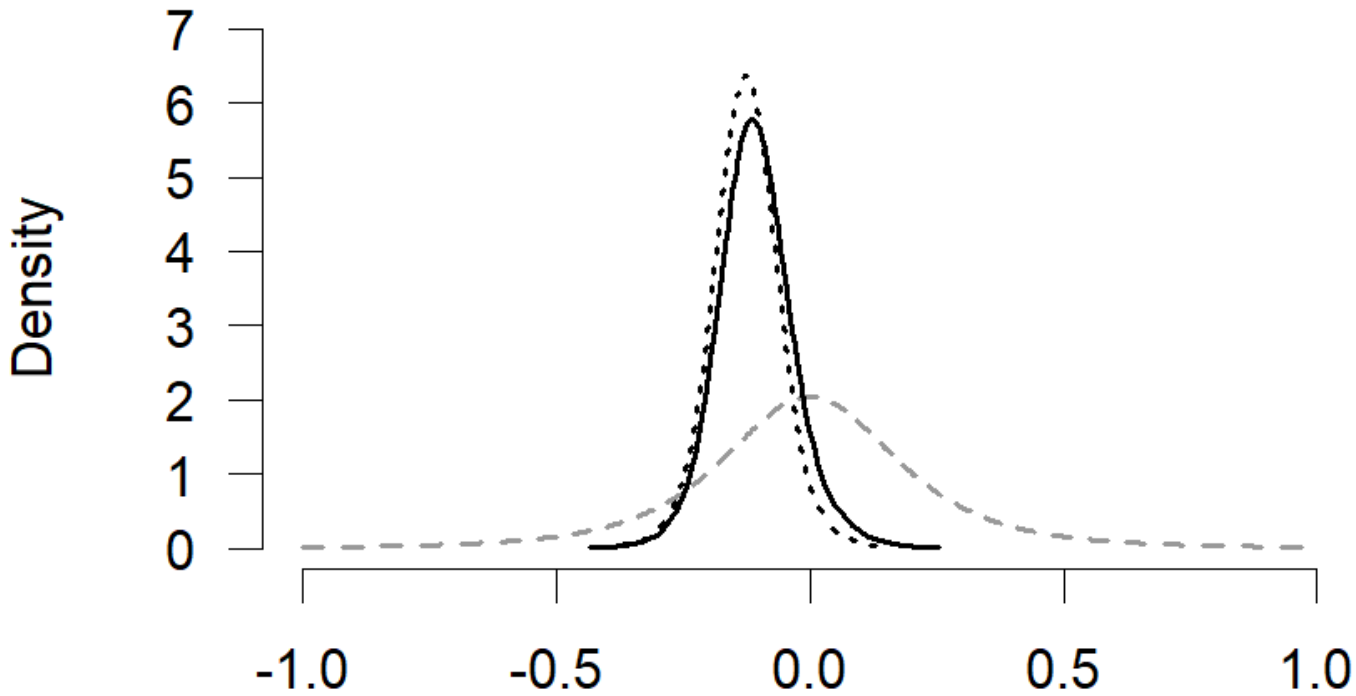
Model-Averaged Prior and Posterior Distribution

```
> plot(fit.RoBMA, parameter = "mu", prior = TRUE, …)
```



Model-Averaged Informed Hypothesis Tests from Density*

```
> hypothesis(fit.RoBMA, c(
    "mu != 0 vs mu = 0",
    "mu  < 0 vs mu = 0",
    "mu  < 0 vs mu > 0"), conditional = TRUE)
 Alternative:    Null:        BF
      mu != 0   mu = 0    1.311
       mu < 0   mu = 0    2.383
       mu < 0   mu > 0    9.953
```

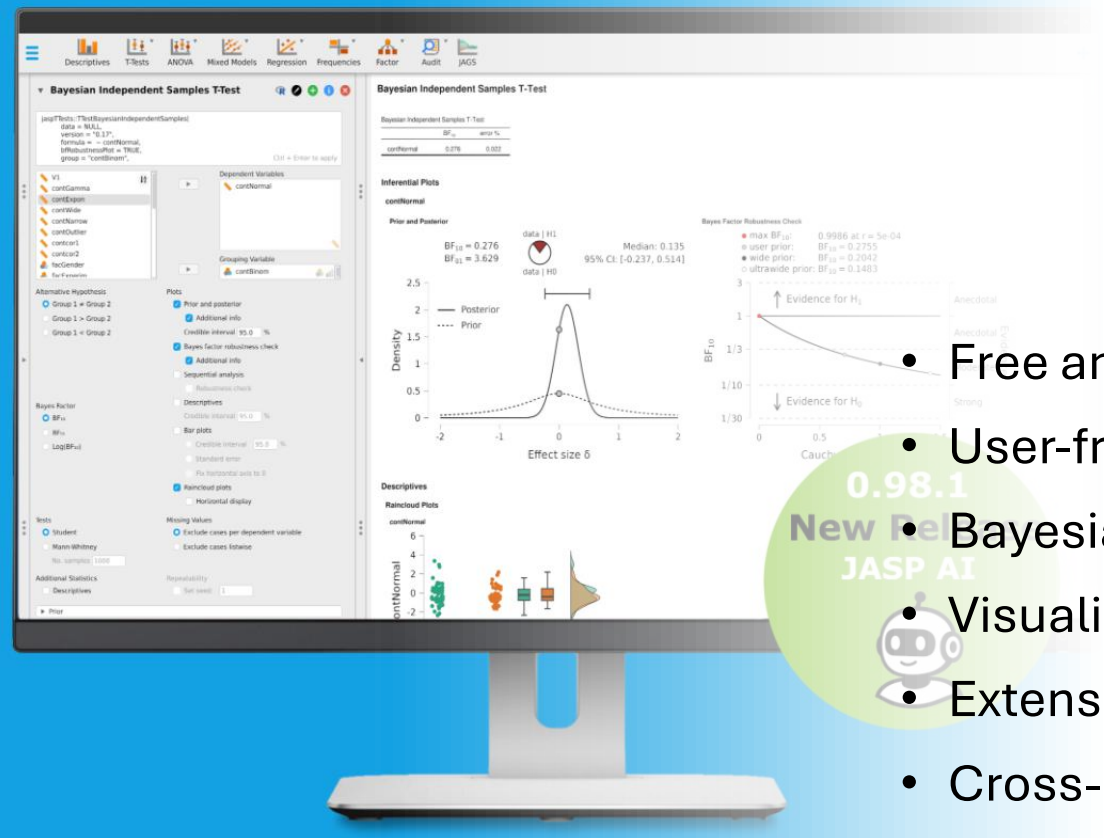


*in the upcoming version of the package

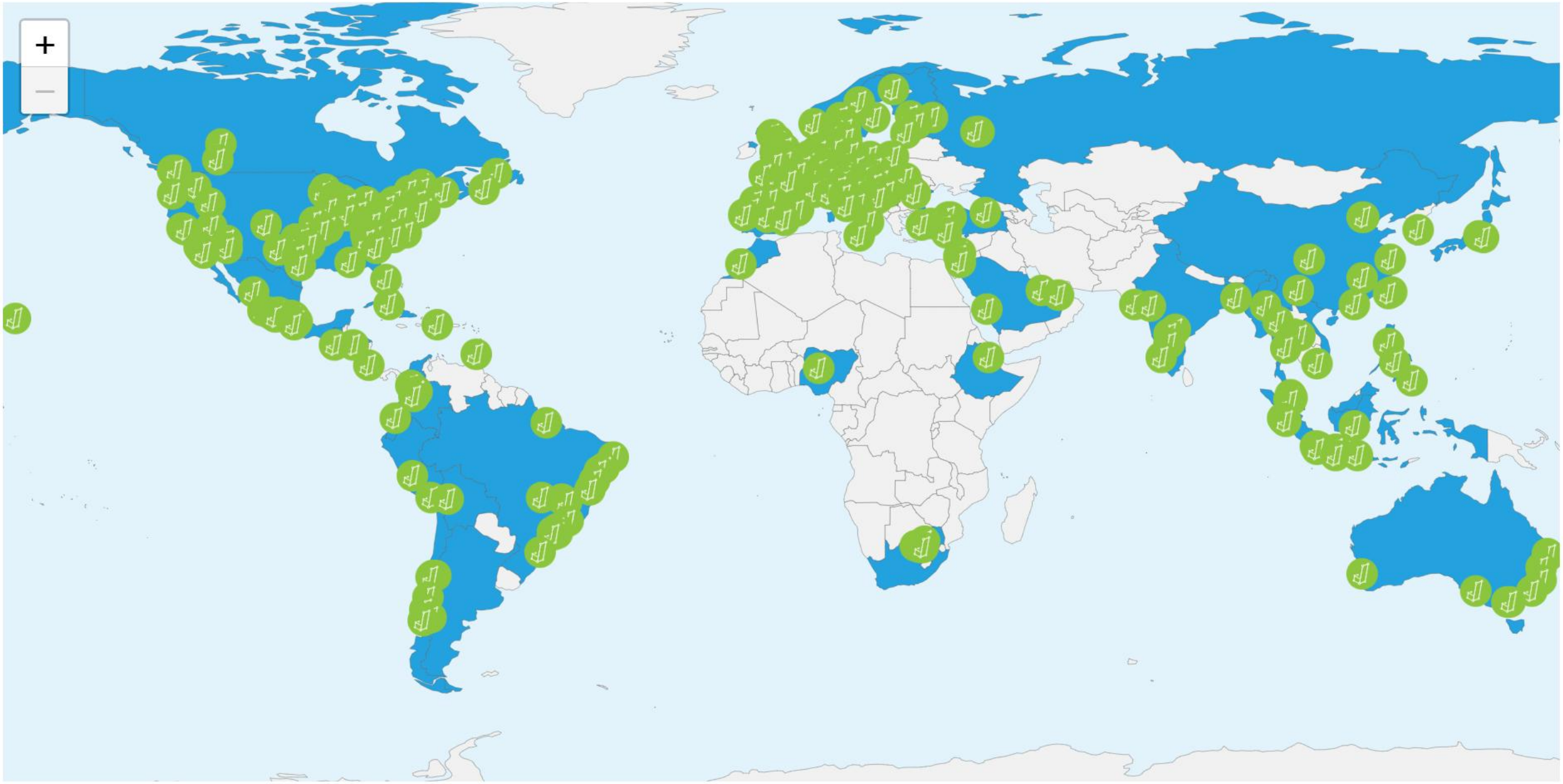
Not everyone wants to code...

A Fresh Way to Do Statistics

Download JASP



- Free and open-source
- User-friendly interface
- Bayesian and frequentist
- Visualization & reporting ready
- Extensibility
- Cross-platform compatibility



374 universities from 76 different countries

Example 2: BCG Vaccines

- Multiple studies examining the efficacy of the Bacillus Calmette–Guérin (BCG) vaccine in preventing tuberculosis
- For each study:
  - number of tuberculosis cases and non-cases in the vaccinated (treatment) condition
  - number of tuberculosis cases and non-cases in unvaccinated (control) condition
  - additional information such as location (via latitude) and randomization procedure

Colditz, G. A., Brewer, T. F., Berkey, C. S., Wilson, M. E., Burdick, E., Fineberg, H. V., & Mosteller, F. (1994). Efficacy of BCG vaccine in the prevention of tuberculosis: Meta-analysis of the published literature. *Journal of the American Medical Association, 271*(9), 698-702. https://doi.org/10.1001/jama.1994.03510330076038

Viechtbauer W, White T, Noble D, Senior A, Hamilton W (2025). *metadat: Meta-Analysis Datasets*. doi:10.32614/CRAN.package.metadat,R package version 1.4-0, https://CRAN.R-project.org/package=metadat.

Effectiveness of the BCG Vaccine Against Tuberculosis*

Edit Data | Descriptives | T-Tests | ANOVA | Mixed Models | Regression | Frequencies | Factor | Machine Learning | Meta-Analysis | Quality Control | Reliability | Survival

	trial	author	year	tpos	tneg	cpos	cneg	ablat	alloc	computed effect size	computed standard
1	1	Aronson	1948	4	119	11	128	44	random	-0.9386941409	0.5975993242
2	2	Ferguson & Simes	1949	6	300	29	274	55	random	-1.666190729	0.4562152931
3	3	Rosenthal et al	1960	3	228	11	209	42	random	-1.386294361	0.6583411564
4	4	Hart & Sutherland	1977	62	13536	248	12619	52	random	-1.456443549	0.1425286391
5	5	Frimodt-Moller et al	1973	33	5036	47	5761	13	alternate	-0.2191410857	0.2279293253
6	6	Stein & Aronson	1953	180	1361	372	1079	44	alternate	-0.9581220408	0.0995252004
7	7	Vandiviere et al	1973	8	2537	10	619	19	random	-1.633775838	0.4764553234
8	8	TPT Madras	1980	505	87886	499	87892	13	random	0.01202060145	0.06330056881
9	9	Coetzee & Berjak	1968	29	7470	45	7232	27	random	-0.4717460358	0.2386988144
10	10	Rosenthal et al	1961	17	1699	65	1600	42	systematic	-1.401210139	0.2746301628
11	11	Comstock et al	1974	186	50448	141	27197	18	systematic	-0.3408496464	0.1119157443
12	12	Comstock & Webster	1969	5	2493	3	2338	33	systematic	0.4466346823	0.7308639904
13	13	Comstock et al	1976	27	16886	29	17825	33	systematic	-0.01734187393	0.2676473749

Effectiveness of the BCG Vaccine Against Tuberculosis*

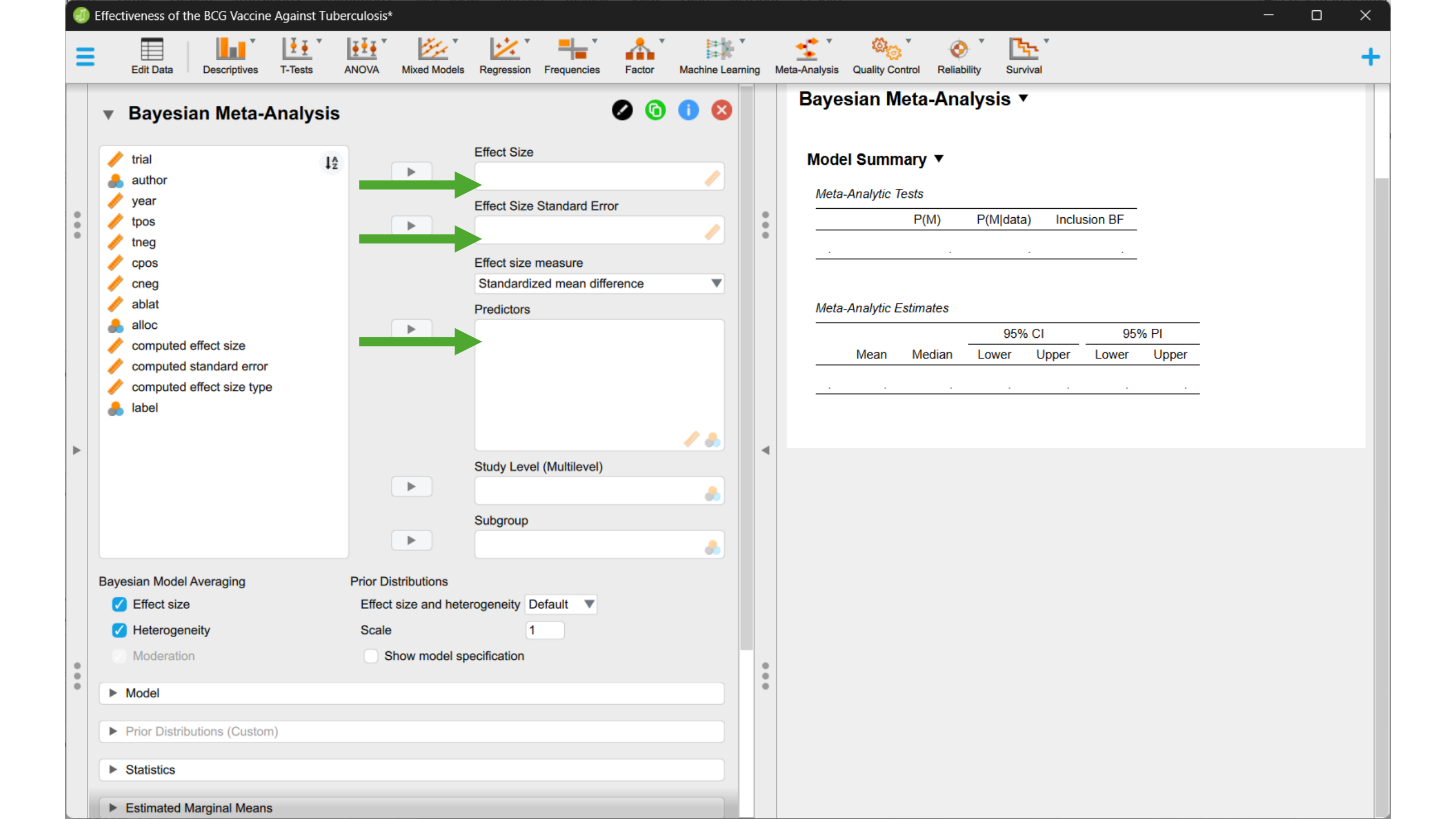
Edit Data | Descriptives | T-Tests | ANOVA | Mixed Models | Regression | Frequencies | Factor | Machine Learning | Meta-Analysis | Quality Control | Reliability | Survival

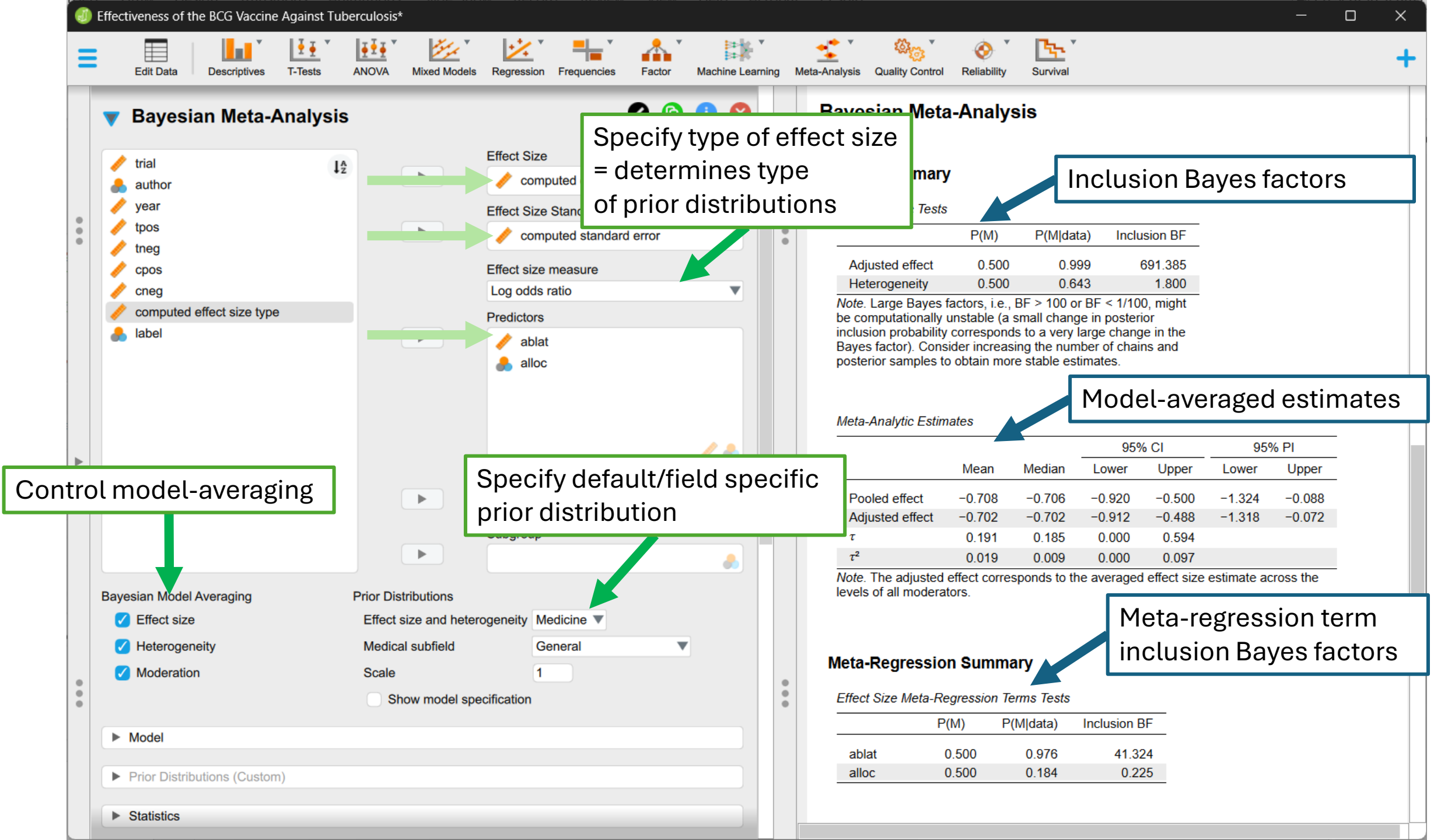
	trial	author	year		tpos	tneg		cpos		computed effect size	computed standard
1	1	Aronson	1948	1,948	4	119		11	12	-0.9386941409	0.5975993242
2	2	Ferguson & Simes	1949	1,949	6	300		29	27	-1.666190729	0.4562152931
3	3	Rosenthal et al	1960	1,960	3	228		11	20	-1.386294361	0.6583411564
4	4	Hart & Sutherland	1977	1,977	62	13536	13,536	248	12	-1.456443549	0.1425286391
5	5	Frimodt-Moller et al	1973	1,973	33	5036	5,036	47	57	-0.2191410857	0.2279293253
6	6	Stein & Aronson	1953	1,953	180	1361	1,361	372	10	-0.9581220408	0.0995252004
7	7	Vandiviere et al	1973	1,973	8	2537	2,537	10	61	-1.633775838	0.4764553234
8	8	TPT Madras	1980	1,980	505	87886	87,886	499	87	0.01202060145	0.06330056881
9	9	Coetzee & Berjak	1968	1,968	29	7470	7,470	45	72	-0.4717460358	0.2386988144
10	10	Rosenthal et al	1961	1,961	17	1699	1,699	65	16	-1.401210139	0.2746301628
11	11	Comstock et al	1974	1,974	186	50448	50,448	141	27	-0.3408496464	0.1119157443
12	12	Comstock & Webster	1969	1,969	5	2493	2,493	3	23	0.4466346823	0.7308639904
13	13	Comstock et al	1976	1,976	27	16886	16,886	29	17	-0.01734187393	0.2676473749

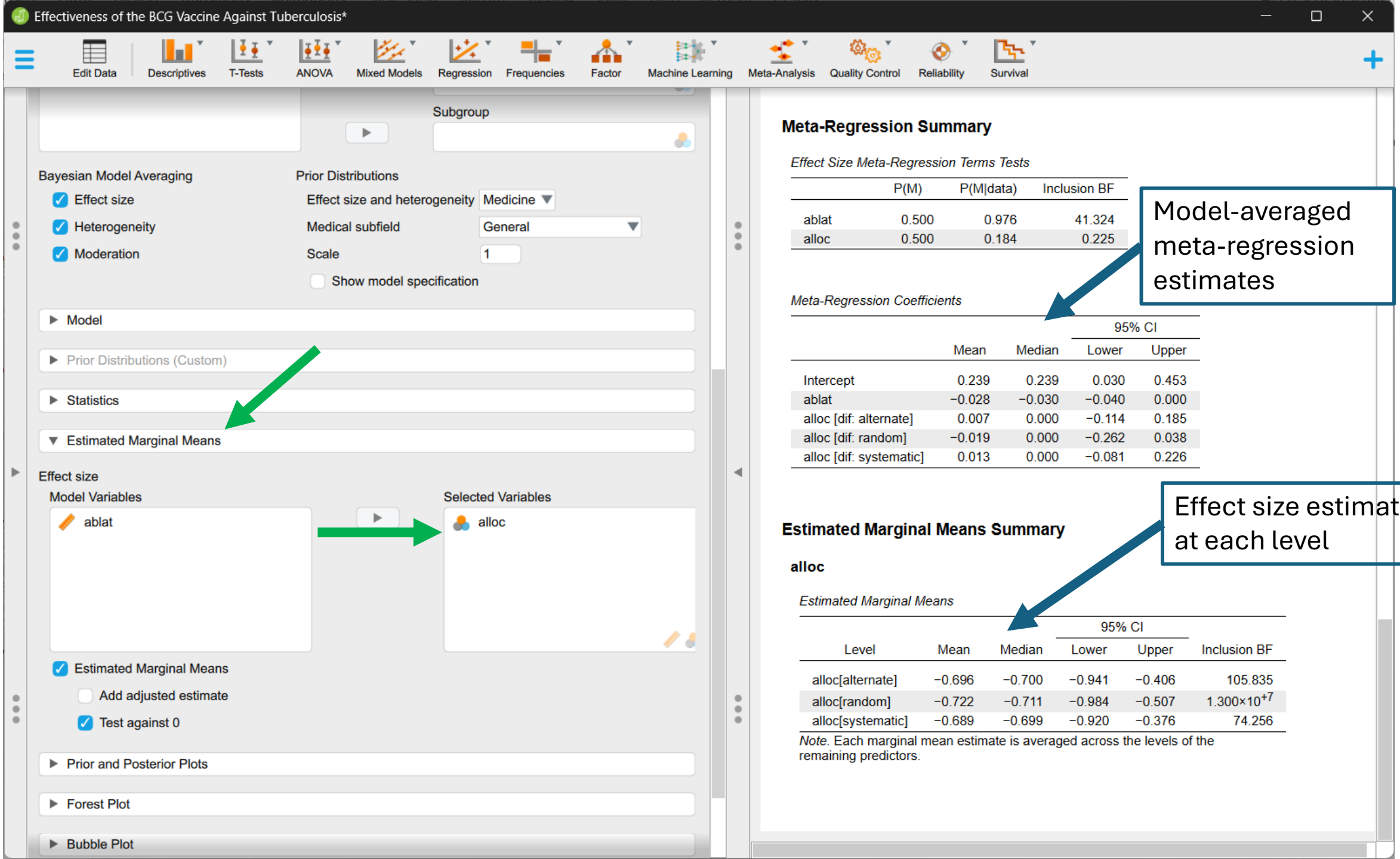
Miscellaneous
- Effect Size Computation
- Effect Size Aggregation
- Funnel Plot
- Forest Plot
- Risk of Bias Plot

Classical
- Meta-Analysis
- Multilevel/Multivariate Meta-Analysis
- Generalized (GLMM) Meta-Analysis
- Mantel-Haenszel / Peto
- Prediction Model Performance
- WAAP-WLS
- PET-PEESE
- Selection Models
- Meta-Analytic SEM
- SEM-Based Meta-Analysis

Bayesian
- Meta-Analysis
- Meta-Analysis (Deprecated)
- Binomial Meta-Analysis
- Penalized Meta-Analysis
- Prediction Model Performance
- Robust Bayesian Meta-Analysis


Effectiveness of the BCG Vaccine Against Tuberculosis*
Edit Data
Descriptives
T-Tests
ANOVA
Mixed Models
Regression
Frequencies
Factor
Machine Learning
Meta-Analysis
Quality Control
Reliability
Survival
Bayesian Meta-Analysis
trial
author
year
tpos
tneg
cpos
cneg
ablat
alloc
computed effect size
computed standard error
computed effect size type
label
Effect Size
Effect Size Standard Error
Effect size measure
Standardized mean difference
Predictors
Study Level (Multilevel)
Subgroup
Bayesian Model Averaging
Effect size
Heterogeneity
Moderation
Prior Distributions
Effect size and heterogeneity
Default
Scale
1
Show model specification
Model
Prior Distributions (Custom)
Statistics
Estimated Marginal Means
Bayesian Meta-Analysis
Model Summary
Meta-Analytic Tests
P(M)
P(M|data)
Inclusion BF
Meta-Analytic Estimates
95% CI
95% PI
Mean
Median
Lower
Upper
Lower
Upper


Effectiveness of the BCG Vaccine Against Tuberculosis*
Edit Data
Descriptives
T-Tests
ANOVA
Mixed Models
Regression
Frequencies
Factor
Machine Learning
Meta-Analysis
Quality Control
Reliability
Survival
Bayesian Meta-Analysis
trial
author
year
tpos
tneg
cpos
cneg
computed effect size type
label
Effect Size
computed
Effect Size Stand
computed standard error
Effect size measure
Log odds ratio
Predictors
ablat
alloc
Specify type of effect size = determines type of prior distributions
Control model-averaging
Specify default/field specific prior distribution
Bayesian Model Averaging
Effect size
Heterogeneity
Moderation
Prior Distributions
Effect size and heterogeneity
Medicine
Medical subfield
General
Scale
1
Show model specification
Model
Prior Distributions (Custom)
Statistics
Bayesian Meta-Analysis
mary
Tests
P(M)
P(M|data)
Inclusion BF
Adjusted effect 0.500 0.999 691.385
Heterogeneity 0.500 0.643 1.800
Note. Large Bayes factors, i.e., BF > 100 or BF < 1/100, might be computationally unstable (a small change in posterior inclusion probability corresponds to a very large change in the Bayes factor). Consider increasing the number of chains and posterior samples to obtain more stable estimates.
Inclusion Bayes factors
Model-averaged estimates
Meta-Analytic Estimates
95% CI
95% PI
Mean
Median
Lower
Upper
Lower
Upper
Pooled effect −0.708 −0.706 −0.920 −0.500 −1.324 −0.088
Adjusted effect −0.702 −0.702 −0.912 −0.488 −1.318 −0.072
τ 0.191 0.185 0.000 0.594
τ² 0.019 0.009 0.000 0.097
Note. The adjusted effect corresponds to the averaged effect size estimate across the levels of all moderators.
Meta-regression term inclusion Bayes factors
Meta-Regression Summary
Effect Size Meta-Regression Terms Tests
P(M)
P(M|data)
Inclusion BF
ablat 0.500 0.976 41.324
alloc 0.500 0.184 0.225


Effectiveness of the BCG Vaccine Against Tuberculosis*
Edit Data
Descriptives
T-Tests
ANOVA
Mixed Models
Regression
Frequencies
Factor
Machine Learning
Meta-Analysis
Quality Control
Reliability
Survival
Subgroup
Bayesian Model Averaging
Effect size
Heterogeneity
Moderation
Prior Distributions
Effect size and heterogeneity
Medicine
Medical subfield
General
Scale
1
Show model specification
Model
Prior Distributions (Custom)
Statistics
Estimated Marginal Means
Effect size
Model Variables
ablat
Selected Variables
alloc
Estimated Marginal Means
Add adjusted estimate
Test against 0
Prior and Posterior Plots
Forest Plot
Bubble Plot
Meta-Regression Summary
Effect Size Meta-Regression Terms Tests
P(M)
P(M|data)
Inclusion BF
ablat 0.500 0.976 41.324
alloc 0.500 0.184 0.225
Model-averaged meta-regression estimates
Meta-Regression Coefficients
95% CI
Mean
Median
Lower
Upper
Intercept 0.239 0.239 0.030 0.453
ablat −0.028 −0.030 −0.040 0.000
alloc [dif: alternate] 0.007 0.000 −0.114 0.185
alloc [dif: random] −0.019 0.000 −0.262 0.038
alloc [dif: systematic] 0.013 0.000 −0.081 0.226
Effect size estimate at each level
Estimated Marginal Means Summary
alloc
Estimated Marginal Means
95% CI
Level
Mean
Median
Lower
Upper
Inclusion BF
alloc[alternate] −0.696 −0.700 −0.941 −0.406 105.835
alloc[random] −0.722 −0.711 −0.984 −0.507 1.300×10+7
alloc[systematic] −0.689 −0.699 −0.920 −0.376 74.256
Note. Each marginal mean estimate is averaged across the levels of the remaining predictors.

Effectiveness of the BCG Vaccine Against Tuberculosis*

Edit Data | Descriptives | T-Tests | ANOVA | Mixed Models | Regression | Frequencies | Factor | Machine Learning | Meta-Analysis | Quality Control | Reliability | Survival

alloc

Study Level (Multilevel)

Subgroup

Bayesian Model Averaging

- [x] Effect size
- [x] Heterogeneity
- [x] Moderation

Prior Distributions

Effect size and heterogeneity: Medicine

Medical subfield: General

Model-averaged prior and posterior distributions

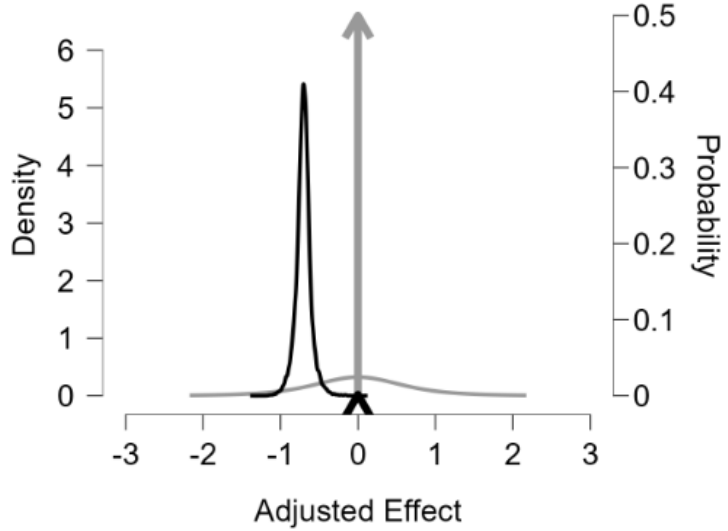
- ▶ Model
- ▶ Prior Distributions (Custom)
- ▶ Statistics
- ▶ Estimated Marginal Means
- ▼ Prior and Posterior Plots

- [x] Effect size
- [x] Heterogeneity
- [ ] Moderation
  - [ ] Estimated marginal means

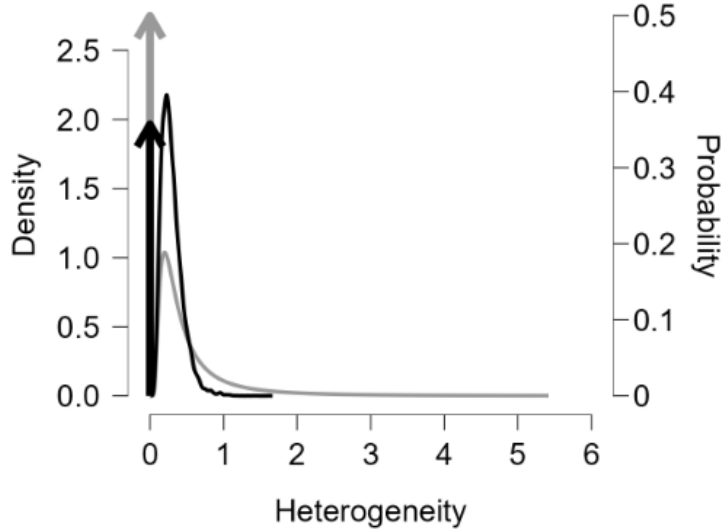
Type

- (•) Model averaged
- ( ) Conditional
- [x] Include prior distribution

- ▶ Forest Plot
- ▼ Bubble Plot



Heterogeneity



Effectiveness of the BCG Vaccine Against Tuberculosis*

Edit Data | Descriptives | T-Tests | ANOVA | Mixed Models | Regression | Frequencies | Factor | Machine Learning | Meta-Analysis | Quality Control | Reliability | Survival

Heterogeneity
Moderation
Estimated marginal means
Model averaged
Conditional
Include prior distribution

▶ Forest Plot

▼ Bubble Plot

Model Variables
alloc

Selected Variable
ablat

Separate Lines

Separate Plots

SD factor covariates 1

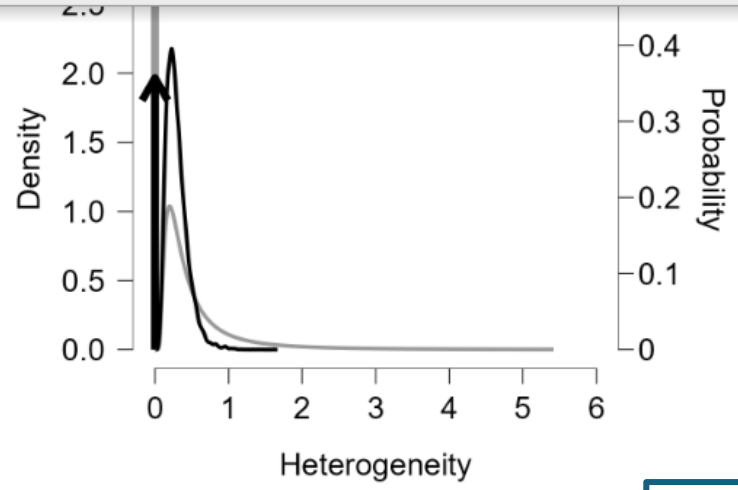
Confidence intervals
Transparency 0.3

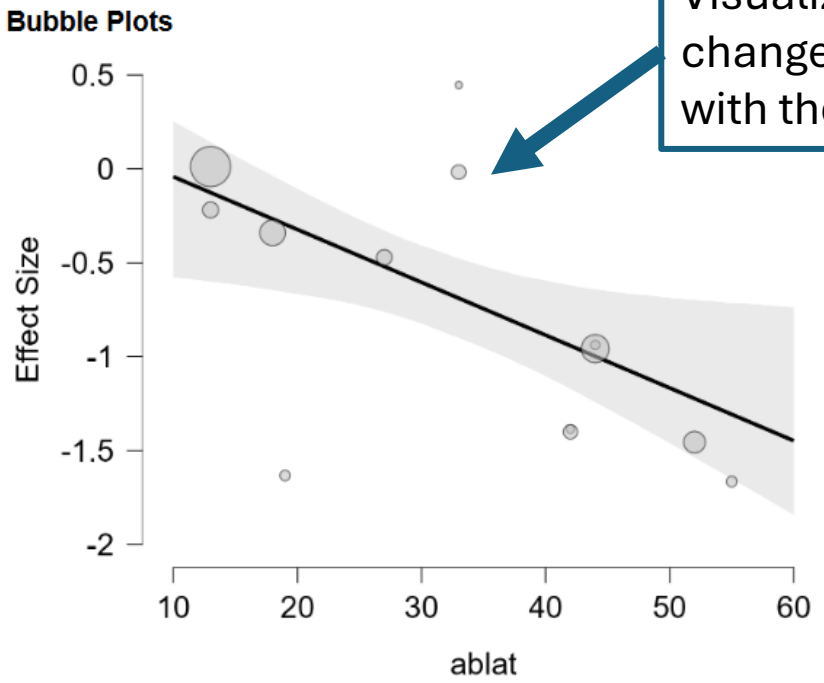
Bubbles
Size Inverse variance
Relative size 1
Transparency 0.9
Jitter 1

Color palette Colorblind
Theme JASP
Relative text size 1.5
Legend position Right

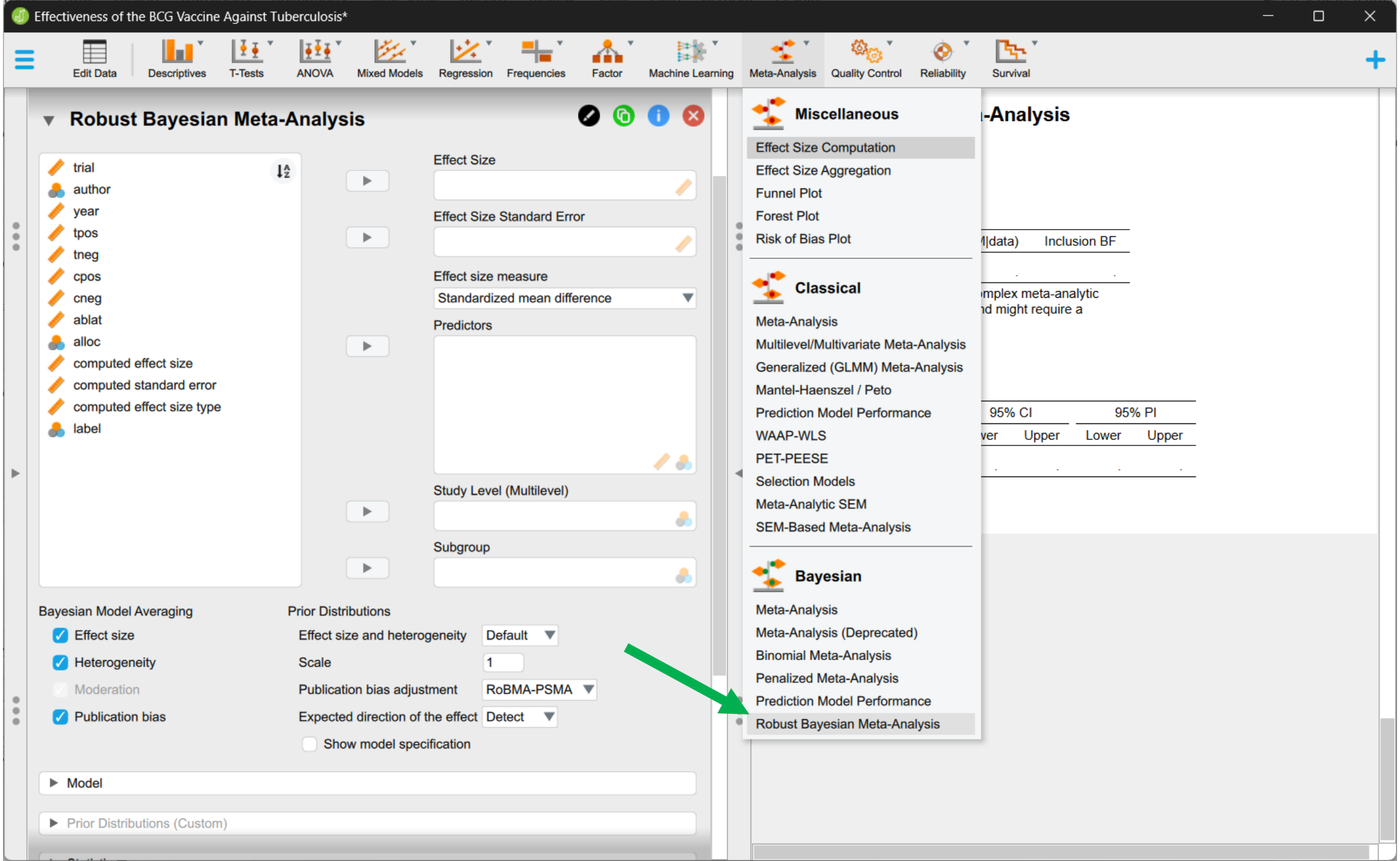
▶ MCMC Diagnostics

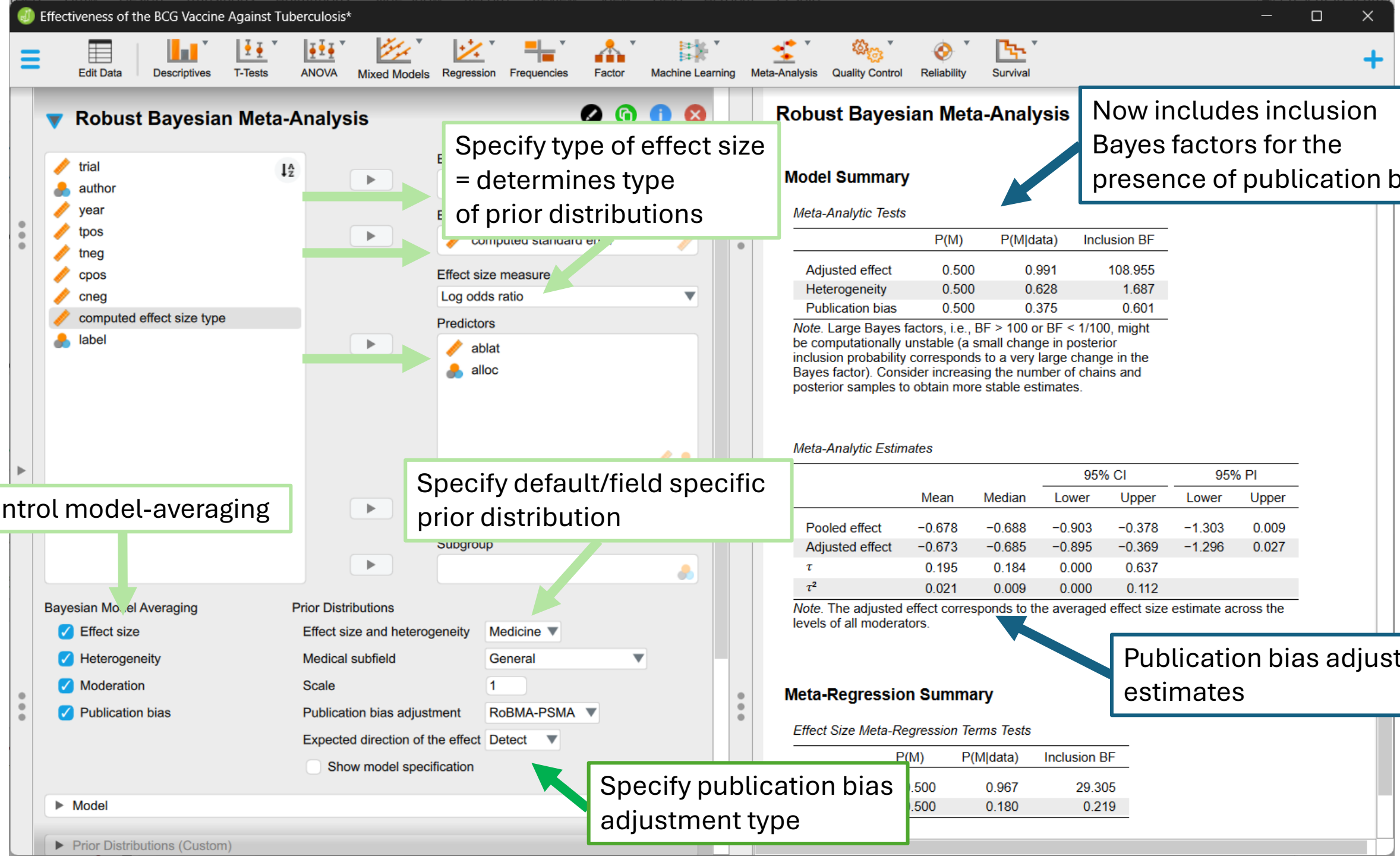
▶ Advanced





Visualizing the change in effect size with the moderator


Effectiveness of the BCG Vaccine Against Tuberculosis*
Edit Data
Descriptives
T-Tests
ANOVA
Mixed Models
Regression
Frequencies
Factor
Machine Learning
Meta-Analysis
Quality Control
Reliability
Survival
Robust Bayesian Meta-Analysis
trial
author
year
tpos
tneg
cpos
cneg
ablat
alloc
computed effect size
computed standard error
computed effect size type
label
Effect Size
Effect Size Standard Error
Effect size measure
Standardized mean difference
Predictors
Study Level (Multilevel)
Subgroup
Bayesian Model Averaging
Effect size
Heterogeneity
Moderation
Publication bias
Prior Distributions
Effect size and heterogeneity
Default
Scale
1
Publication bias adjustment
RoBMA-PSMA
Expected direction of the effect
Detect
Show model specification
Model
Prior Distributions (Custom)
Miscellaneous
Effect Size Computation
Effect Size Aggregation
Funnel Plot
Forest Plot
Risk of Bias Plot
Classical
Meta-Analysis
Multilevel/Multivariate Meta-Analysis
Generalized (GLMM) Meta-Analysis
Mantel-Haenszel / Peto
Prediction Model Performance
WAAP-WLS
PET-PEESE
Selection Models
Meta-Analytic SEM
SEM-Based Meta-Analysis
Bayesian
Meta-Analysis
Meta-Analysis (Deprecated)
Binomial Meta-Analysis
Penalized Meta-Analysis
Prediction Model Performance
Robust Bayesian Meta-Analysis
-Analysis
|data)
Inclusion BF
mplex meta-analytic
nd might require a
95% CI
95% PI
ver
Upper
Lower
Upper


Effectiveness of the BCG Vaccine Against Tuberculosis*
Edit Data
Descriptives
T-Tests
ANOVA
Mixed Models
Regression
Frequencies
Factor
Machine Learning
Meta-Analysis
Quality Control
Reliability
Survival
Robust Bayesian Meta-Analysis
trial
author
year
tpos
tneg
cpos
cneg
computed effect size type
label
Effect size measure
Log odds ratio
Predictors
ablat
alloc
Subgroup
Bayesian Model Averaging
Effect size
Heterogeneity
Moderation
Publication bias
Prior Distributions
Effect size and heterogeneity
Medicine
Medical subfield
General
Scale
1
Publication bias adjustment
RoBMA-PSMA
Expected direction of the effect
Detect
Show model specification
Model
Prior Distributions (Custom)
Specify type of effect size = determines type of prior distributions
Control model-averaging
Specify default/field specific prior distribution
Specify publication bias adjustment type
Robust Bayesian Meta-Analysis
Model Summary
Meta-Analytic Tests
P(M) P(M|data) Inclusion BF
Adjusted effect 0.500 0.991 108.955
Heterogeneity 0.500 0.628 1.687
Publication bias 0.500 0.375 0.601
Note. Large Bayes factors, i.e., BF > 100 or BF < 1/100, might be computationally unstable (a small change in posterior inclusion probability corresponds to a very large change in the Bayes factor). Consider increasing the number of chains and posterior samples to obtain more stable estimates.
Meta-Analytic Estimates
95% CI 95% PI
Mean Median Lower Upper Lower Upper
Pooled effect −0.678 −0.688 −0.903 −0.378 −1.303 0.009
Adjusted effect −0.673 −0.685 −0.895 −0.369 −1.296 0.027
τ 0.195 0.184 0.000 0.637
τ² 0.021 0.009 0.000 0.112
Note. The adjusted effect corresponds to the averaged effect size estimate across the levels of all moderators.
Meta-Regression Summary
Effect Size Meta-Regression Terms Tests
P(M) P(M|data) Inclusion BF
.500 0.967 29.305
.500 0.180 0.219
Now includes inclusion Bayes factors for the presence of publication bias
Publication bias adjusted estimates

Features

- Standard meta-analysis: brma / brma.glmm / brma.mv*
- Publication bias: bselmodel / bselmodel.mv* / bPET / bPET.mv* / ...
- Bayesian model-averaging: BMA / BMA.glmm / BMA.mv*
- Robust Bayesian meta-analysis: RoBMA / RoBMA.mv*
- Model fit: marginal likelihood, loo
- Model diagnostics: residuals, posterior predictives
- Model manipulation: predict, blup, ranef, ...

... https://fbartos.github.io/RoBMA/articles/v03-feature-coverage.html

*in the upcoming version of the package

Thank You For Your Attention

Accessible and easy to use software is necessary for adoption of new approaches.

Bartoš, F., & Wagenmakers, E. J. (2025). Meta-analysis with JASP, Part II: Bayesian approaches. *arXiv preprint arXiv:2509.09850*.

https://jasp-stats.org/

https://cran.r-project.org/web/packages/RoBMA